1D:

$$-\frac{d^2}{dx^2}\sin(kx) = k^2\sin(kx) = \left(\frac{2\pi}{\lambda}\right)^2\sin(kx) = \left(\frac{2\pi p}{h}\right)^2\sin(kx) = \frac{p^2}{(h/2\pi)^2}\sin(kx) = \frac{p^2}{\hbar^2}\sin(kx)$$

$$-\hbar^2\frac{d^2}{dx^2}\sin(kx) = p^2\sin(kx)$$

1D: $\frac{p^2}{2m} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

3D:

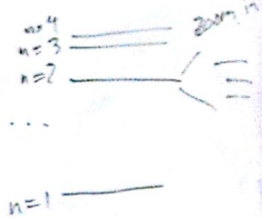
$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\sin(k_x x + k_y y + k_z z) = \underbrace{(k_x^2+k_y^2+k_z^2)}_{= k^2 = \left(\frac{2\pi}{\lambda}\right)^2 = \frac{p^2}{\hbar^2}}\sin(k_x x + k_y y + k_z z) \Rightarrow \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$$

Schrodinger equation for H atom:

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) - \frac{e^2}{4\pi\varepsilon_0\sqrt{x^2+y^2+z^2}}\right]\Psi(x,y,z) = E\Psi(x,y,z)$$

Like with particle-in-a-box, only certain energy levels occur.

$$E \approx \frac{-13.6\,eV}{n^2} \qquad n = 1, 2, 3, 4, 5, \ldots$$



$$1\,eV = (1.60\times10^{-19}\,C)\left(1\,\frac{J}{C}\right) = 1.60\times10^{-19}\,J$$

$$\frac{p^2}{2m} + U = E \qquad \left(\frac{p^2}{2m} + U\right)\psi = E\psi$$

kinetic + potential = total energy

Hydrogen atom: $\psi(x,y,z)$ wave function

(x,y,z) = position of electron relative to nucleus

$$\frac{h}{p} = \lambda \qquad k = \frac{2\pi}{\lambda}$$

$$U = \frac{-e^2}{4\pi\varepsilon_0 r} \qquad r = \sqrt{x^2+y^2+z^2}$$

1D wave $A\sin(kx)$

3D wave $A\sin\left(k_x x + k_y y + k_z z\right)$ $\qquad k = \sqrt{k_x^2 + k_y^2 + k_z^2}$

In general, $\psi(x,y,z) = \sum_{k_x,k_y,k_z} A_{k_x,k_y,k_z} \sin(k_x x + k_y y + k_z z)$

Transition from state (n, ℓ, m_ℓ, m_s) to $(n', \ell', m'_\ell, m'_s)$:

emit/absorb photon with $E_{photon} = \dfrac{hc}{\lambda_{photon}} = \left| E_{n,\ell,m_\ell,m_s} - E_{n',\ell',m'_\ell,m'_s} \right|$

E depends mostly on n & ℓ;
m_ℓ & m_s make relatively tiny contributions.

Z = # protons in nucleus

$$E_{n,\ell} \approx -13.6\,eV \cdot \frac{Z_{eff}^2}{n^2}$$

Z_{eff} = effective nuclear charge $Z_{eff} = 1$ for H

$Z_{eff} \le Z$.

$$\underbrace{E_{1,0}}_{\substack{\text{row 1}\\ 1 \le Z \le 2}} < \underbrace{E_{2,0} < E_{2,1}}_{\substack{\text{row 2}\\ 3 \le Z \le 4}} < \underbrace{E_{3,0} < E_{3,1}}_{\text{row 3}} < \underbrace{E_{4,0} < E_{3,2} < E_{4,1}}_{\text{row 4}} < \underbrace{E_{5,0} < E_{4,2} < E_{5,1}}_{\text{row 5}} < \underbrace{E_{6,0} < E_{4,3} \lesssim E_{5,2} < E_{6,1}}_{\text{row 6}} < \underbrace{E_{7,0} < E_{5,3}}_{\text{row 7}}$$

$E_{1,0}$	$E_{2,0}$	$E_{2,1}$	$E_{3,0}$	$E_{3,1}$	$E_{4,0}$	$E_{3,2}$	$E_{4,1}$	$E_{5,0}$	$E_{4,2}$	$E_{5,1}$	$E_{6,0}$	$E_{4,3}$	$E_{5,2}$	$E_{6,1}$	$E_{7,0}$	$E_{5,3}$
2	2	6	2	6	2	10	6	2	10	6	2	14	10	6	2	14

not observed/synthesized yet

$$\ldots \lesssim E_{6,2} < \underbrace{E_{7,1}}_{113 \le Z \le 118} < E_{8,0} < \ldots$$

$E_{6,2}$	$E_{7,1}$	$E_{8,0}$
10	6	2

ℓ	$4\ell + 2$
0	2
1	6
2	10
3	14

ground state:

$n=1$ solution: $\psi(x,y,z) = \frac{1}{\sqrt{\pi r_0^3}} e^{-r/r_0}$

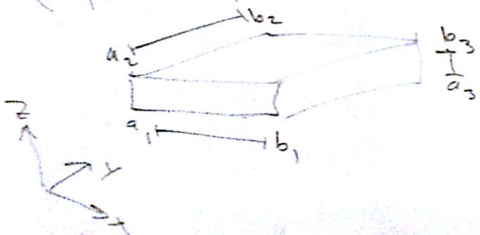
Bohr radius

$r_0 = 5.29 \times 10^{-11}$ m

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\psi|^2 \, dz\, dy\, dx = 1$$

$$\int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} |\psi|^2 \, dz\, dy\, dx$$

$$\text{Prob}(a_1 \le x \le b_1 \ \& \ a_2 \le y \le b_2 \ \& \ a_3 \le z \le b_3)$$



$$\text{Prob}(r_1 \le r \le r_2) = \int_{r_1}^{r_2} |\psi|^2 4\pi r^2 \, dr$$

for H ground state

$L = \sqrt{\ell(\ell+1)}\,\hbar$

orbital angular momentum

$\ell = 0, 1, 2, \ldots, n-1$

$L_z = m_\ell \hbar$

$\underbrace{m_\ell = \pm\ell \ \pm(\ell-1), \ldots, \pm 2, \pm 1, 0}_{2\ell+1 \text{ values}}$

$S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\,\hbar = \sqrt{3}\,\hbar/2$

spin angular momentum of electron

$s=\frac{1}{2}$: electrons have spin $\frac{1}{2}$

photons have spin 1 & angular momentum $\hbar$

$S_z = \pm\frac{1}{2}\hbar = m_s \hbar$ $\quad \underbrace{m_s = \pm\frac{1}{2}}_{2 \text{ values}}$

For a given n & ℓ, there are $4\ell+2$ electron states.