$$\lim_{x \to -2^+} \underbrace{\frac{5\ln(2+x)+2x+3}{7-8\ln(2+x)}}_{f(x)} = -\frac{5}{8} = -0.625$$

x	$f(x)$
$-2+10^{-1}=-1.9$	$-0.4843\ldots$
$-2+10^{-2}=-1.99$	$-0.5475\ldots$
$-2+10^{-3}=-1.999$	$-0.5707\ldots$
$-2+10^{-4}=-1.9999$	$-0.5831\ldots$
$-2+10^{-5}=-1.99999$	$-0.5909\ldots$
$-2+10^{-6}-1.999999$	-0.5962
$-2+10^{-7}$	$-0.6001\ldots$
$-2+10^{-8}$	$-0.6031\ldots$
$-2+10^{-20}$	$-0.6160\ldots$
$-2+10^{-30}$	$-0.6189\ldots$
$-2+10^{-50}$	$-0.6213\ldots$
$-2+10^{-100}$	$-0.6231\ldots$
$-2+10^{-1000}$	$-0.6248\ldots$
$-2+10^{-10000}$	$-0.6249\ldots$
$-2+10^{-100000}$	$-0.62499\ldots$

It's too bad your calculator rounds off $-2+10^{-20}$ and all later x-values on the list to -2. When naive calculator experiments fail...

...try hyperreals!

Let $0 < \varepsilon \approx 0$ and $x = (-2)+\varepsilon \; (=\varepsilon-2)$.

$$f(-2+\varepsilon) = \frac{5\ln\varepsilon + 2(\varepsilon-2)+3}{7-8\ln\varepsilon}$$

$-H = \ln\varepsilon$ is ~~positive~~ negative infinite.

$$f(-2+\varepsilon) = \frac{-5H+2\varepsilon-4+3}{7+8H}$$

$$f(-2+\varepsilon) = \frac{-5H+2\varepsilon-1}{7+8H}$$

"Divide" by the largest "term" in the denominator:

$$f(-2+\varepsilon) = \frac{(-5H+2\varepsilon-1)/H}{(7+8H)/H}$$

$$f(-2+\varepsilon) = \frac{-5+2\varepsilon/H-1/H}{7/H+8}$$

$$st(f(-2+\varepsilon)) = \frac{-5+0-0}{0+8} = -\frac{5}{8}$$

So, the limit is $-5/8$, which equals -0.625.

I think it's always faster to use hyperreals first. I only use a calculator to double-check.