$$\lim_{x \to 0} \frac{7+(3/x)}{2-(4/x)} = \; ?$$

$$0 \neq \varepsilon \approx 0$$

$$st\left(\frac{7+3/(0+\varepsilon)}{2-4/(0+\varepsilon)}\right) = st\left(\frac{7+3/\varepsilon}{2-4/\varepsilon}\right)$$

$$= st\left(\frac{[7+3/\varepsilon]\cdot\varepsilon}{[2-4/\varepsilon]\cdot\varepsilon}\right) = st\left(\frac{7\varepsilon+3}{2\varepsilon-4}\right)$$

$$= \frac{7\cdot 0+3}{2\cdot 0-4} = \frac{0+3}{0-4} = -\frac{3}{4}$$

$$\Rightarrow \lim_{x \to 0} \frac{7+(3/x)}{2-(4/x)} = \boxed{-\frac{3}{4}}$$

$$\lim_{y\to 3^+}\left(\ln\left(\frac{1}{\sqrt{y-3}}\right)+4\right) = ? \quad \text{Try:}$$

$$st\left(\ln\left(\frac{1}{\underset{\substack{\uparrow\\ 0<\varepsilon\approx 0}}{\sqrt{3+\varepsilon-3}}}+4\right)\right) = st\left(\ln\left(\frac{1}{\sqrt{\varepsilon}}+4\right)\right) =$$

$0<\sqrt{\varepsilon}\approx 0$, so $\frac{1}{\sqrt{\varepsilon}}$ is infinite & positive,

so $\frac{1}{\sqrt{\varepsilon}}+4$ is infinite & positive,

so $\ln\left(\frac{1}{\sqrt{\varepsilon}}+4\right)$ is infinite & positive.

$\Rightarrow st\left(\ln\left(\frac{1}{\sqrt{\varepsilon}}+4\right)\right)$ does not exist

$\Rightarrow \lim_{y\to 3^+}\ln\left(\frac{1}{\sqrt{y-3}}+4\right)$ $\boxed{\text{does not exist}}$

$$\lim_{y \to 4} 2\ln\left(\frac{1}{\sqrt{y-3}}\right) = ?$$

$$st\left(2\ln\left(\frac{1}{\sqrt{4+\underset{\substack{\uparrow \\ 0 \neq \varepsilon \simeq 0}}{\varepsilon}-3}}\right)\right)$$

$$= 2\ln\frac{1}{\sqrt{4+0-3}} = 2\ln\frac{1}{\sqrt{1}} = 2\ln 1$$

$$= 2 \cdot 0 = 0$$

$$\Rightarrow \lim_{y \to 4} 2\ln\left(\frac{1}{\sqrt{y-3}}\right) = \boxed{0}$$

$$\lim_{z \to 0^+} \log_2 z = \;?$$

$0 < \varepsilon \approx 0$

$$st\left(\log_2(0+\varepsilon)\right) = st\left(\frac{\ln(0+\varepsilon)}{\ln 2}\right) = \frac{st(\ln \varepsilon)}{\ln 2}$$

But $\ln \varepsilon$ is infinite & negative, so

$st(\ln \varepsilon)$ does not exist.

Moreover, $\dfrac{\ln \varepsilon}{\ln 2}$ is infinite, so

$$\lim_{z \to 0^+} \log_2 z \quad \boxed{\text{D.N.E.}}$$

$$\lim_{z \to 0^+} \frac{\log_4(8z)}{1+\log_5 z} = ?$$

$$st\left(\frac{\log_4(8(0+\varepsilon))}{1+\log_5(\underset{\substack{\uparrow \\ 0<\varepsilon\approx 0}}{0+\varepsilon})}\right) = st\left(\frac{\log_4(8\varepsilon)}{1+\log_5 \varepsilon}\right)$$

$$= st\left(\frac{\log_4 8 + \log_4 \varepsilon}{1+\log_5 \varepsilon}\right) = st\left(\frac{\log_4 8 + (\ln \varepsilon)/\ln 4}{1+\underbrace{(\ln \varepsilon)}/\ln 5}\right)$$

$\ln \varepsilon$ is negative & infinite

$$= st\left(\frac{[\log_4 8 + (\ln \varepsilon)/\ln 4]/\ln \varepsilon}{[1+\ln \varepsilon]/\ln \varepsilon}\right)$$

$$= st\left(\frac{\frac{\log_4 8}{\ln \varepsilon} + \frac{1}{\ln 4}}{\frac{1}{\ln \varepsilon} + \frac{1}{\ln 5}}\right) = \left(\frac{0+\frac{1}{\ln 4}}{0+\frac{1}{\ln 5}}\right)$$

$$= \frac{\ln 5}{\ln 4} \Rightarrow \lim_{z \to 0^+} \frac{\log_4(8z)}{1+\log_5 z} = \boxed{\frac{\ln 5}{\ln 4}}$$

$$\lim_{x \to 1^+} 5^{\frac{x+1}{x-1}} = ?$$

$$0 < \varepsilon \approx 0$$

$$st\left(5^{\frac{1+\varepsilon+1}{1+\varepsilon-1}}\right) = st\left(5^{(2+\varepsilon)/\varepsilon}\right)$$

$$= st\left(5^{(2/\varepsilon)+(\varepsilon/\varepsilon)}\right) = st\left(5^{2/\varepsilon}\, 5^{1}\right)$$

$$= 5\ st\left(5^{2/\varepsilon}\right)$$

$$= 5\ st\left(e^{(2/\varepsilon)\ln 5}\right) = ?$$

$\frac{2}{\varepsilon}$ is infinite & positive; $\ln 5$ is positive & non-infinitesimal. So, $\frac{2}{\varepsilon} \ln 5$ is infinite & positive. So, $e^{(2/\varepsilon)\ln 5}$ is infinite

$$\Rightarrow \lim_{x \to 1^+} 5^{(x+1)/(x-1)} \quad \boxed{D.N.E.}$$

$$\lim_{v \to 0^+} \frac{5 + \sqrt{v}}{1 + \ln v} = ?$$

$0 < \varepsilon \simeq 0$

$$st\left(\frac{5 + \sqrt{0+\varepsilon}}{1 + \ln(0+\varepsilon)}\right) = st\left(\frac{5 + \sqrt{\varepsilon}}{1 + \ln \varepsilon}\right) = ?$$

$\ln \varepsilon$ is negative & infinite, so
$1 + \ln \varepsilon$ is negative & infinite.
$5 + \sqrt{\varepsilon}$ is finite. Therefore,
$\frac{5+\sqrt{\varepsilon}}{1+\ln\varepsilon}$ is infinitely small.

$$\Rightarrow st\left(\frac{5 + \sqrt{\varepsilon}}{1 + \ln \varepsilon}\right) = 0$$

$$\Rightarrow \lim_{v \to 0^+} \frac{5 + \sqrt{v}}{1 + \ln v} = \boxed{0}$$

$$\lim_{x\to 0^+}(1+x)^{1/x} = ?$$

$0 < \varepsilon \approx 0$

$$st\left((1+0+\varepsilon)^{1/(0+\varepsilon)}\right) = st\left((1+\varepsilon)^{1/\varepsilon}\right)$$

$$= st\left(e^{(\ln(1+\varepsilon))/\varepsilon}\right) = e^{st((\ln(1+\varepsilon))/\varepsilon)}$$

$$st\left(\frac{\ln(1+\varepsilon)}{\varepsilon}\right) = st\left(\frac{\ln(1+\varepsilon) - \overbrace{\ln 1}^{0}}{\varepsilon}\right) = f'(1)$$

where $f(x) = \ln x$.

$f'(x) = 1/x$

$$\Rightarrow \lim_{x\to 0^+}(1+x)^{1/x} = e^{f'(1)} = e^{1/1} = \boxed{e}$$

$$\lim_{x \to -1} \log_3\left(\frac{1}{(x+1)^4} + 1\right) = ?$$

$$st\left(\log_3\left(\frac{1}{(-1+\varepsilon+1)^4} + 1\right)\right) = st\left(\log_3\left(\frac{1}{\varepsilon^4} + 1\right)\right)$$

$0 \neq \varepsilon \approx 0$

$$= st\left(\frac{\ln(1/\varepsilon^4 + 1)}{\ln 3}\right) = ?$$

$\frac{1}{\varepsilon^4}$ is $\left\{\begin{matrix}\text{infinite \&} \\ \text{positive}\end{matrix}\right\} \Rightarrow \frac{1}{\varepsilon^4} + 1$ is $\left\{\begin{matrix}\text{infinite \&} \\ \text{positive}\end{matrix}\right\}$

$\Rightarrow \ln\left(\frac{1}{\varepsilon^4} + 1\right)$ is $\left\{\begin{matrix}\text{infinite \&} \\ \text{positive}\end{matrix}\right\}$

$\Rightarrow \frac{\ln\left(\frac{1}{\varepsilon^4} + 1\right)}{\ln 3}$ is $\left\{\begin{matrix}\text{infinite \&} \\ \text{positive}\end{matrix}\right\}$

$$\Rightarrow \lim_{x \to -1} \log_3\left(\frac{1}{(x+1)^4} + 1\right) \quad \boxed{\text{D.N.E.}}$$

$$\lim_{u\to 0} \frac{e^{1/u^2}}{3-5^{1/u^2}} = ?$$

$0 \neq \varepsilon \approx 0$

$$st\left(\frac{e^{1/(0+\varepsilon)^2}}{3-5^{1/(0+\varepsilon)^2}}\right) = st\left(\frac{e^{1/\varepsilon^2}}{3-\underbrace{e^{(\ln 5)/\varepsilon^2}}}\right)$$

$$= st\left(\frac{e^{1/\varepsilon^2}/e^{(\ln 5)/\varepsilon^2}}{\left(3-e^{(\ln 5)/\varepsilon^2}\right)/e^{(\ln 5)/\varepsilon^2}}\right)$$

$$= st\left(\frac{e^{(1-\ln 5)/\varepsilon^2}}{3/\left(e^{(\ln 5)/\varepsilon^2}\right)-1}\right) = ?$$

$\ln 5 > 0$

$\Downarrow$

$(\ln 5)/\varepsilon^2$ is positive & infinite

$\Downarrow$

$e^{(\ln 5)/\varepsilon^2}$ is positive & infinite

$\frac{3}{e^{(\ln 5)/\varepsilon^2}}$ is infinitesimal (≈ 0)

$\ln 5 > \ln e = 1$, so $\ln 5 > 1$, so $1 - \ln 5 < 0$, so $\frac{1 - \ln 5}{\varepsilon^2}$ is negative & infinite, so $e^{((1 - \ln 5)/\varepsilon^2)}$ is ≈ 0.

Therefore, $\cancel{\lim_{u \to 0} \frac{e^{1/u^2}}{3 - 5^{1/u^2}}}$

$$st\left(\frac{e^{(1-\ln 5)/\varepsilon^2}}{3/(e^{(\ln 5)/\varepsilon^2}) - 1}\right) = \frac{0}{0 - 1} = 0$$

$$\Rightarrow \lim_{u \to 0} \frac{e^{1/u^2}}{3 - 5^{1/u^2}} = \boxed{0}$$