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Section 1.6 - Standard Parts 08/31/10 pg. 1

Standard = real #s
Non Standard = hyperreals

Team	Meaning
1.) b finite	$a < b < c$ for some a, c real
2.) H positive + infinite	$a < H$ for all real a
3.) H negative + infinite	$H < a$ for all real a
4.) infinitesimal (ε)	$-a < \varepsilon < a$ for all real $a > 0$
5.) x infinitely close to y	$x - y$ is infinitesimal

Standard Part Principle:
Every finite hyperreal b is in infinitely close to some real number.
→ we'll call that real the standard part of "b", written $\underline{st(b)}$.

So, if b is finite, then $\underbrace{b \approx st(b)}_{\text{infinitely close}}$

(i.e. $b - st(b)$ is infinitesimal)

If I let $\varepsilon = b - st(b)$, then $b = st(b) + \varepsilon$.

Ex) If δ is infinitesimal, then $2\delta - 7 \approx -7$.
You can't be infinitely close to two different reals.

Ex) x real | b hyperreal | y real

→ If $b - x$ and $y - b$ are infinitesimal, then $(b - x) + (y - b)$ is infinitesimal too.

But $(b-x)+(y-b) = y-x$, so $x \approx y$.
x, y are real, so $y-x$ is real and infinitesimal, so $y-x = 0$.

* Let $a + b$ be finite hyperreals:

$st(-a) = -st(a)$

$st(a+b) = st(a) + st(b)$

$st(ab) = st(a) \cdot st(b)$

$st(a/b) = st(a)/st(b)$ if $st(b) \neq 0$

$st(a^n) = st(a)^n$

$st(\sqrt[n]{a}) = \sqrt[n]{st(a)}$ if $a \geq 0$

$st(a-b) = st(a) - st(b)$

If $a \leq b$, then $st(a) \leq st(b)$

If Δx is infinitesimal and x is real, find $st(3x(x+\Delta x)^2 - \Delta x)$.

$$\begin{aligned}
st(3x(x+\Delta x)^2 - \Delta x) &= st(3x(x+\Delta x)^2) - \underset{0}{\underline{st(\Delta x)}} \\
&= st(3)\,st(x)\,st((x+\Delta x)^2) - \\
&= 3x(st(x+\Delta x)^2) - 0 \\
&= 3x(st(x) + st(\Delta x))^2 - 0 \\
&= 3x(x+0)^2 - 0 \\
&= 3x^3
\end{aligned}$$

Let $\varepsilon \neq 0$ and ε be infinitesimal.

$$st\left(\frac{1}{5\varepsilon} - \frac{1}{5\varepsilon + \varepsilon^2}\right) = ?$$

*do NOT break it down because you would divide by 0, which cannot be done.

Answer: $st\left(\frac{1}{5\varepsilon} - \frac{1}{5\varepsilon+\varepsilon^2}\right) = st\left(\frac{1}{5\varepsilon} - \frac{1}{\varepsilon(5+\varepsilon)}\right)$

$$= st\left(\frac{1}{5\varepsilon}\cdot\frac{5+\varepsilon}{5+\varepsilon} - \frac{1}{\varepsilon(5+\varepsilon} \cdot \frac{5}{5}\right) = st\left(\frac{5+\varepsilon}{5\varepsilon(5+\varepsilon)} - \frac{5}{5\varepsilon(5+\varepsilon}\right.$$

$$= st\left(\frac{\cancel{\varepsilon}}{5\cancel{\varepsilon}(5+\varepsilon)}\right) = st\left(\frac{1}{5(5+\varepsilon)}\right)$$

$$= \frac{1}{5(5+st(\varepsilon))} = \frac{1}{5(5+0)} = \frac{1}{25}$$

$$st\left(\frac{\sqrt{7+\delta} - \sqrt{7}}{\delta}\right) = ? \quad \text{if } \begin{cases}\delta \approx 0 \\ \delta \neq 0\end{cases}$$

*Idea: $(a-b)(a+b) = a^2 - b^2$

$a, b \geq 0 \Rightarrow (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2$

$= a - b$

Answer: $st\left(\frac{\sqrt{7+\delta} - \sqrt{7}}{\delta}\right) = st\left(\frac{(\sqrt{7+\delta} - \sqrt{7})(\sqrt{7+\delta} + \sqrt{7})}{\delta(\sqrt{7+\delta} + \sqrt{7})}\right)$

$$st\left(\frac{7+\delta-7}{\delta(\sqrt{7+\delta}+\sqrt{7})}\right) = st\left(\frac{\cancel{\delta}}{\cancel{\delta}(\sqrt{7+\delta}+\sqrt{7})}\right) = st\left(\frac{1}{\sqrt{7+\delta}+\sqrt{7}}\right)$$

$$= \frac{1}{\sqrt{7+0}+\sqrt{7}} = \frac{1}{2\sqrt{7}}$$