September 9th Notes

Today 2.3 Part I

Product Rule $\qquad$ Improved Power Rule

$$y=(x^3+x^2+1)(5x^5-\sqrt{x}) = \underline{5x^8+5x^7+5x^6+5x^5-x^{7/2}-x^{5/2}-x^{3/2}-\sqrt{x}}$$

$$y' = \frac{dy}{dx}$$

There's an easier way using Product rule.

Product Rule:

1) $(fg)' = f'g + fg'$ $\qquad$ 2) $\dfrac{d(fg)}{dx} = \dfrac{df}{dx}\,g + f\,\dfrac{dg}{dx}$

3) $d(fg) = (df)g + f\,dg$

$d(fg) = g\,df + f\,dg$

$$y' = \frac{dy}{dx} = (x^3+x^2+x+1)'(5x^5-\sqrt{x}) + (x^3+x^2+x+1)(5x^5-\sqrt{x})' =$$
$$=(3x^2+2x+1)(5x^5-\sqrt{x}) + (x^3+x^2+x+1)\left(5\cdot 5x^4 - \frac{1}{2\sqrt{x}}\right)$$

Picture Sample

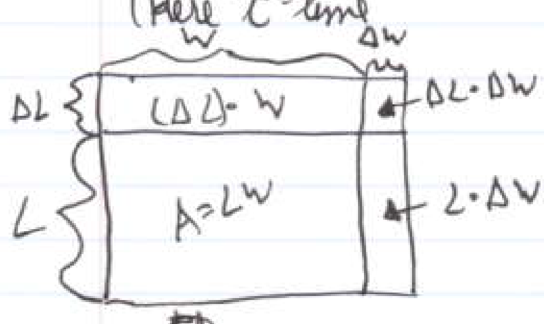
A rectangle has length L and width W

area $= A = LW$

If L and W are changing (at rate $\frac{dL}{dt}$ + $\frac{dW}{dt}$),

then what is $\frac{dA}{dt}$, the rate of change of area?

(Here t = time)



$$\Delta A = (\Delta L)W + (\Delta L)\Delta W + L\Delta W$$

$$\frac{\Delta A}{\Delta t} = \frac{\Delta L}{\Delta t}W + \frac{\Delta L}{\Delta t}\Delta W + L\frac{\Delta W}{\Delta t}$$

$$\frac{dA}{dt} = st\left(\frac{\Delta A}{\Delta t}\right) = \frac{dL}{dt}\Big(W + \overbrace{st(\Delta W)}^{0}\Big) + L\frac{dW}{dt} = \boxed{\frac{dL}{dt}W + L\frac{dW}{dt}}$$

$\Delta W \approx 0$ (by increment theorem)

Primes: $A' = L'W + LW'$

E.g. if $L = 5\text{ cm}$, $W = 8\text{ cm}$, $\frac{dL}{dt} = \frac{.2\text{ cm}}{s}$ & $\frac{dW}{dt} = .3\frac{\text{cm}}{s}$

then $\frac{dA}{dt} = (.2)(8) + (5)(.3) = 3.1\,\frac{\text{cm}^2}{s}$

One possibility: $t = 0$

& $L = 5 + .2t + t^2$ $\quad \frac{dL}{dt} = .2 + 2t$

$W = 8 + .3t - t^3$ $\quad \frac{dW}{dt} = .3 - 3t^2$

$$\frac{dA}{dt} = \frac{d(LW)}{dt} = \frac{dL}{dt}W + L\frac{dW}{dt} = (.2+2t)(8+.3t-t^3) + (5+.2t+t^2)(.3-3t$$

$$z = (w^3 - \sqrt{w})(5w^2 + 2/w)$$

$$\frac{dz}{dw} = (w^3 - \sqrt{w})'(5w^2 + 2/w) + (w^3 - \sqrt{w})(5w^2 + 2/w)'$$

$$3w^2 - \frac{1}{2\sqrt{w}}\,(5w^2 + 2/w) + (w^3 - \sqrt{w})(10w - 2/w^2)' \quad \& \quad \left(\frac{1}{w}\right)' = \frac{1}{w^2}$$

Product Rule

$y = (x^2+1)^{17}$ $\cdot$ $y' = 17(x^2+1)^{16}(x^2+1)'$

$= 17(x^2+1)^{16}(2x)$

$s = (t - 3\sqrt{t})^5$. $\frac{ds}{dt} = 5(t - 3\sqrt{t})^4 (t - 3\sqrt{t})'$

$= 5(t - 3\sqrt{t})^4\left(1 - \frac{3}{2\sqrt{t}}\right)$

$d(fg) = f\,dg + g\,df$

* $d(f^2) = d(ff) = f\,df + f\,df = 2f\,df$

$d(f^3) = f\,d(f^2) + f^2\,df$

$= f(2f\,df) + f^2\,df$

$= 2f^2\,df + f^2\,df = 3f^2\,df$

$d(f^n) = nf^{n-1}\,df$

$(f^n)' = \frac{d f^n}{dx} = nf^{n-1}\frac{df}{dx} = nf^{n-1}f'$

Assuming x independent variable

$(f^n)' = nf^{n-1}f'$ (Improved power rule)

Product Rule: $(fg)' = f'g + fg'$

Less Common: $(fgh)' = f'gh + fg'h + fgh'$

$abcd = a'bcd + ab'cd + abc'd + abcd'$

Ex proof $(fgh)' = ((fg)h)' = (fg)'h + (fg)h' = \underbrace{(f'g + fg')h}_{f'gh + fg'h} + fgh'$

$u = [(t^2-1)^3](5+t^4)$

$\frac{du}{dt} = [(t^2-1)^3]'(5+t^4) + (t^2-1)^3(5+t^4)'$

$= 3(t^2-1)^2(t^2-1)'(5+t^4) + (t^2-1)^3(0+4t^3)$

$= 3(t^2-1)^2(2t-0)(5+t^4) + (t^2-1)^3(4t^3)$