May Tumonong
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Suppose $x = g(y) = y^5 + 3y - 7$

Notice that $\underline{g'(y) = 5y^4 + 3} - 0 = 5\underbrace{(y^2)^2}_{\geq 0} + 3 \geq 5 \cdot 0 + 3 = 3$

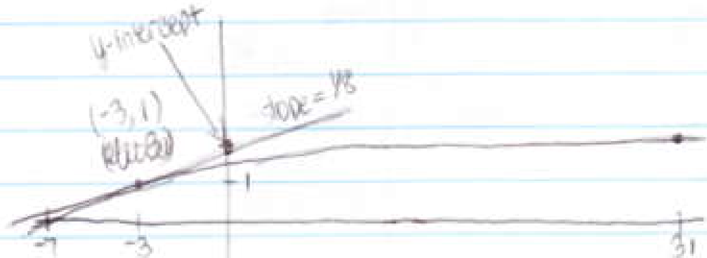
$\Rightarrow g'(y) \neq 0$ everywhere

$\Rightarrow$ By the Inverse Function Theorem,
g has an inverse f & f' exists & $f' \neq 0$

$x = g(y)$ & f is the inverse of g, so $y = f(x)$
There is no formula for f: solving $x = y^5 + 3y - 7$ is too hard

When $y = 1$, $x = 1^5 + 3 \cdot 1 - 7 = 1 + 3 - 7 = -3$ We can find $f'(-3)$
Inverse Function Rule: $f'(x) = \dfrac{1}{g'(y)} = \dfrac{1}{5y^4 + 3}$

$f'(-3) = \dfrac{1}{5 \cdot 1^4 + 3} = \dfrac{1}{5+3} = \dfrac{1}{8}$ (slope)



$x = g(y) = y^5 + 3y - 7$
$y = 0 \Rightarrow x = -7$
$y = 2 \Rightarrow x = 31$

<u>Equation for tangent line:</u>

~~$y - (-3) = \frac{1}{8}(x - 1)$~~
~~same as~~
~~$y = \frac{1}{8}(x-1) - 3$~~

$y - 1 = \frac{1}{8}(x - (-3))$
$y = \frac{1}{8}(x+3) + 1$
When $x = 0$
$y = \frac{1}{8}(0+3) + 1$
$y = \frac{11}{8}$

$$(\sqrt[n]{x})' = (x^{1/n})' = \boxed{\tfrac{1}{n} x^{\frac{1}{n}-1}}$$

↑ Why?

Inverse functions
$$\begin{cases} y = x^{1/n} \\ y = (x^{1/n})^n = x^{1/n \cdot n} = x^1 = x \end{cases}$$

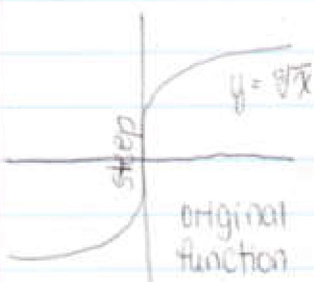
$$(x^{1/n})' = \frac{1}{(y^n)'} = \frac{1}{ny^{n-1}} = \frac{1}{n} \cdot \frac{1}{y^{n-1}} = \frac{1}{n} \cdot y^{-(n-1)} = \frac{1}{n} y^{1-n}$$

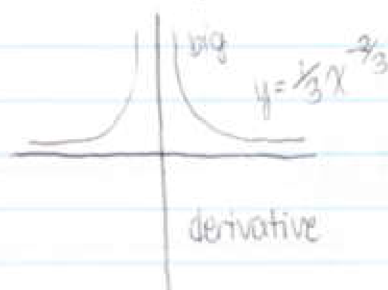
$$= \frac{1}{n}(x^{1/n}) = \frac{1}{n} x^{1/n \cdot (1-n)} = \frac{1}{n} x^{1/n - n/n} = \boxed{\frac{1}{n} x^{1/n - 1}}$$

$$(\sqrt[3]{x})' = (x^{1/3}) = \tfrac{1}{3} x^{1/3 - 1} = \boxed{\tfrac{1}{3} x^{-2/3}}$$

$$= \frac{1}{3}(x^{-1/3})^2 = \frac{1}{3}\left(\frac{1}{x^{1/3}}\right)^2 =$$

$$= \frac{1}{3}\left(\frac{1}{\sqrt[3]{x}}\right)^2 = \boxed{\frac{1}{3(\sqrt[3]{x})^2}}$$





If $r = \frac{m}{n}$ is rational like $\left(\frac{2}{3}, \frac{-5}{4}, 7, -2, 0, \frac{-1}{2}, \text{etc}\right)$

then $(x^r)' = r x^{r-1}$ (Rational Power Rule)

Proof: See Section 2.4

$$\left(\frac{5}{\sqrt{x^7}}\right)' = 5\left(\frac{1}{x^{7/2}}\right)' = 5\left(x^{-7/2}\right)'$$

$$= 5\left(\frac{-7}{2}\right) x^{-7/2 - 1}$$

$$= \boxed{\frac{-35}{2} x^{-9/2}}$$