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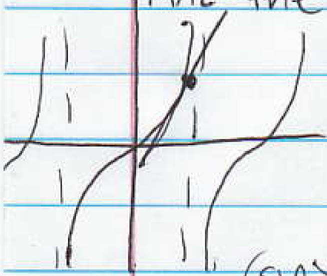
Find the line tangent to $y=\tan x$ at $x=\frac{\pi}{3}$



- Find the point — $x=\pi/3$ $y=\tan\left(\frac{\pi}{3}\right)=\frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)}$
- Find the slope — $(\pi/3, \sqrt{3})$ $\sqrt{3}=\frac{\sqrt{3}/2}{1/2}$
- Point slope formula

$$\left(\frac{\sin x}{\cos x}\right)' = y=\tan x=\frac{\sin x}{\cos x} \leftarrow \begin{cases} \text{Find } \frac{dy}{dx} \\ \text{plug in } x=\pi/3 \end{cases}$$

$$\frac{(\sin x)'\cos x - \sin x(\cos x)'}{\cos^2 x}$$

$$\frac{dy}{dx}=\frac{\cos x\cos x-\sin x(-\sin x)}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$

$$\frac{f'g-fg'}{g^2} \quad \left(\frac{f}{g}\right)'$$

$$\boxed{\sec^2 x}=(\sec x)^2=\left(\frac{1}{\cos x}\right)^2=\boxed{\frac{1}{\cos^2 x}}$$

plug in $\pi/3$ to find slope $=\left(\frac{1}{\cos\frac{\pi}{3}}\right)^2=\left(\frac{1}{1/2}\right)^2=2^2$

slope $=4$

$$\boxed{\begin{aligned} y-\sqrt{3}&=4(x-\pi/3)\\ y&=\sqrt{3}+4(x-\pi/3)\end{aligned}}$$

$a=$ constant >0

$(a^x)'=\boxed{a^x\ln a}$

$(x^n)'=nx^{n-1}$ n a rational constant

$\Delta x=$ non zero infinitesimal

$$st\left(\frac{a^{x+\Delta x}-a^x}{\Delta x}\right)=st\left(\frac{a^x a^{\Delta x}-a^x}{\Delta x}\right)=a^x\, st\left(\frac{a^{\Delta x}-1}{\Delta x}\right)$$

logarithms $=a^x$ & $\log_a x$ are inverse functions

$y=a^x \iff x=\log_a y$

$x=a^{(\log_a x)}=\log_a a^{(a^x)}$

$x \to a^x$, y, $\log_a x$

There is a real $e=2.71828\ldots$ such that if $0\neq\delta\approx0$, then $st\left(\frac{a^\delta-1}{\delta}\right)=\log_e a$ written $\ln a$

$\ln e=\log_e e=\log_e(e^1)$ $\ln e=1$

$(e^x)'=e^x\ln e$

$(e^x)'=e^x$

$y = \log_a x$

$a^y = a^{(\log_a x)} = x$

$y' = (\log_a x)' = \frac{1}{(a^y)'} = \frac{1}{a^y \ln a} = \boxed{\frac{1}{x \ln a}}$

inverse function rule

$(\ln x)' = (\log_e x)' = \frac{1}{\ln e} \quad \boxed{\frac{1}{x}}$

$$\boxed{\begin{aligned} (a^x)' &= a^x \ln a \\ (a^x)' &= e^x \\ (\log_a x)' &= \frac{1}{x \ln a} \\ (\ln x)' &= \frac{1}{x} \end{aligned}}$$

you put \$10,000 into a new savings account which pays you 1.5% interest continuously compounded, meaning if you wait t years the account balance B ~~B~~ $10000(e^{0.015})^t$. How much will the balance be in 2 years? $10000(e^{0.015})^2 = \$10,304.55$

what is the instantaneous rate of change of B after 2 years? 0 years?

$\frac{dB}{dt} = \cancel{10000(e^{0.015})^t}$

$\frac{\Delta y}{\Delta x}$ = average rate of change

$\frac{dy}{dx}$ = instantaneous rate of change

$\frac{dB}{dt} = 10000(e^{0.015}) \frac{\ln}{\log_e} (e^{0.015})$

$\frac{dB}{dt} = 10000(e^{0.015})^t (0.015)$

$dB/dt = 150(e^{0.015})^t$

$\cancel{10000}\ 150 (e^{0.015})^2 = \frac{\$154.57}{year}$ $\quad \frac{\$0.4235}{day}$

$150(e^{0.015})^0$

$150(1)$

$\$150/year$ $\quad \frac{\$0.4107}{day}$

The 30Kg of Plutonium-240 (^{240}Pu) currently in a barrel of radioactive waste. Radioactivity decays with a half-life of $h = 6560$ years, meaning the amount x in kilograms after t years is $30\left(\frac{1}{2}\right)^{(t/6560)}$

$$x = 30\left(\frac{1}{2}\right)^{\frac{t}{6560}} = 30\left(\left(\frac{1}{2}\right)^{\frac{1}{6560}}\right)^{t}$$

$$\frac{dx}{dt} = 30\left(\left(\frac{1}{2}\right)^{\frac{1}{6560}}\right)^{t} \underbrace{\ln\left(\left(\frac{1}{2}\right)^{\frac{1}{6560}}\right)}_{\text{simplify to } \frac{-\ln 2}{6560}}$$

plug in $t = 100$ to $\frac{dy}{dt}$

100 years from now