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Inverse Function Rule:

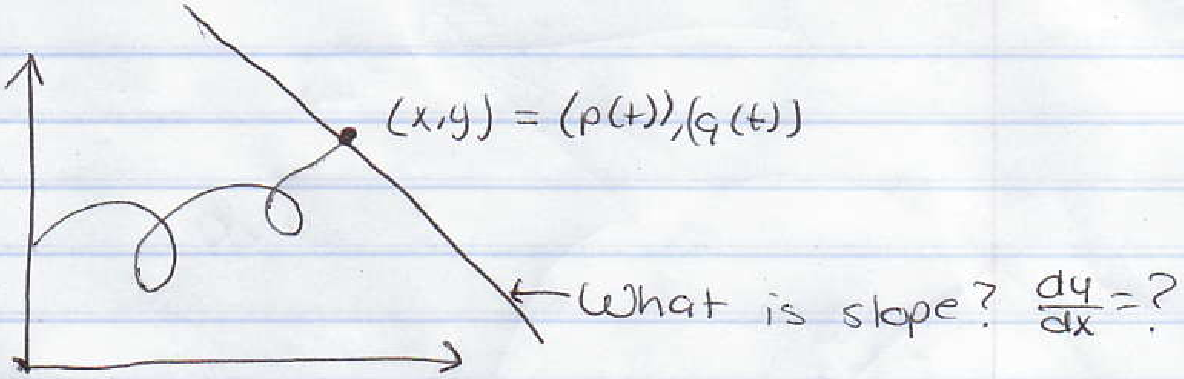
If $y = f(x)$ and $x = g(x)$

$f'(x)$ and $g'(y)$ exists, and $f'(x)$ and $g'(y)$ are non-zeros, then $f'(x) = \frac{1}{g(y)}$

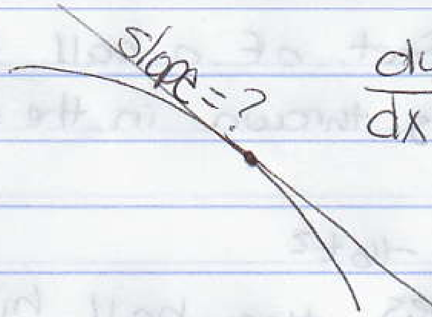
Diffirential version: $dy/dx = \frac{1}{dx/dy}$

Dx's and dy's are different

dy/dx	dx/dy
$y = f(x)$	$x = g(y)$
$dx = \Delta x$	$dy = \Delta y$
$0 \approx \Delta x \neq 0$	$0 \approx \Delta y \neq 0$
$dy = f'(x)dx$	$dx = g'(y)dy$
$\Delta y = f(x+\Delta x) - f(x)$	$\Delta x = g(y+\Delta y) - g(y)$



- Particle in motion: x, y position
  t = time



$$\frac{dy}{dx} = ?$$

A small piece passes the vertical line test, so $y = f(x)$ for some f locally.

$$\underbrace{\frac{dy}{dx} = \frac{dy}{dt}\,\frac{dt}{dx}}_{\text{Chain Rule}} = \frac{dy}{dt}\,\frac{1}{dx/dt} = (dy/dt)\frac{1}{dx/dt} = \frac{dy/dt}{dx/dt}$$

Inverse Function Rule $\rightarrow \frac{dt}{dx} = \frac{1}{dx/dt}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

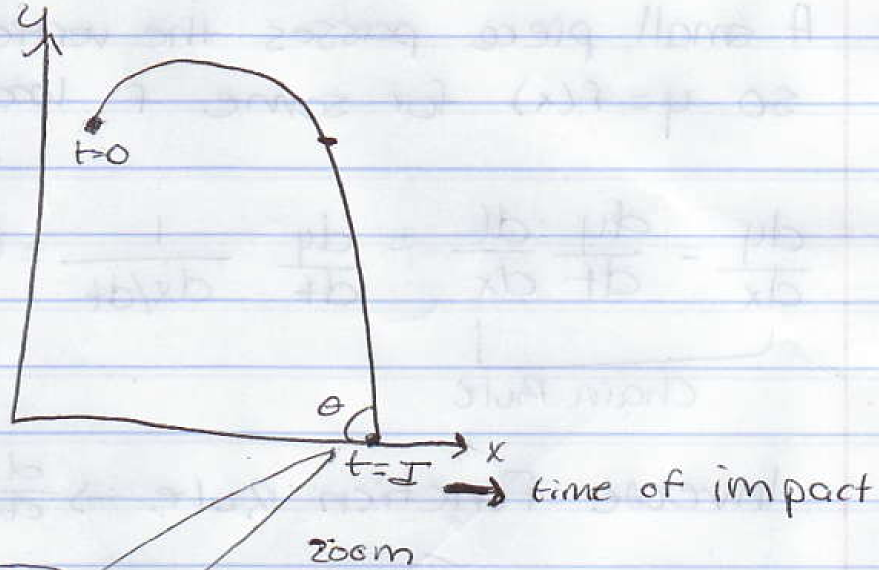
$$\frac{q'(t)}{p'(t)}$$

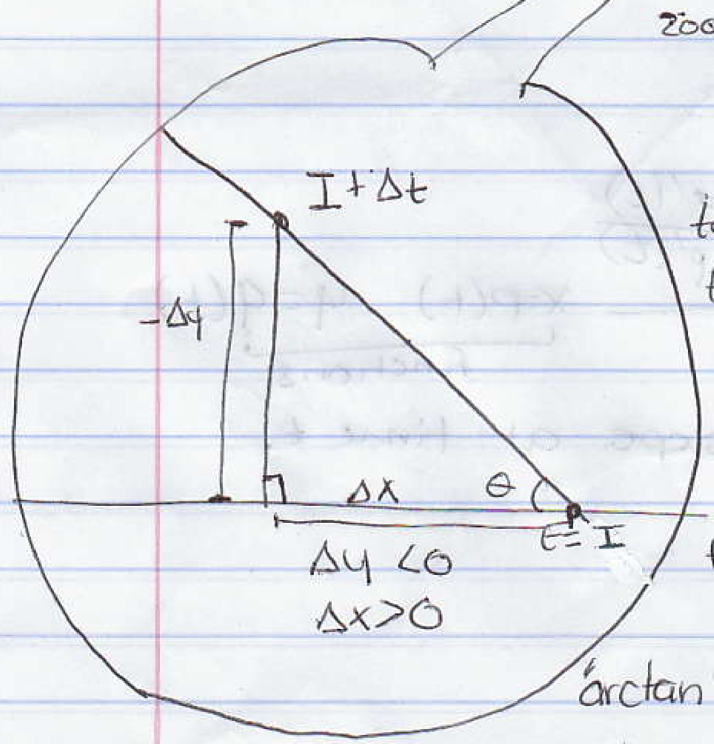
$$\underbrace{\frac{dy}{dx} = \frac{dy/dt}{p'(t)}}_{\text{formula for slope at time } t} \longleftarrow \underbrace{x = p(t) \quad y = q(t)}_{\text{functions}}$$

The (x,y) position in feet of a ball t seconds after being thrown in the air is

$$x(t) = 1+12t$$
$$y(t) = 6+13t-16t^2$$

At what angle does the ball hit the ground $(y=0)$?





$0 \simeq \Delta t < 0$

$\tan\theta \approx \frac{-\Delta y}{\Delta x} \rightarrow \frac{\text{opp.}}{\text{adj.}}$

$\tan\theta$ is real, so

$\tan\theta = st\left(-\left(\frac{\Delta y}{\Delta x}\right)\right) = -\frac{dy}{dx}$ at $t=I$

$\theta = \underbrace{\arctan}_{\text{inverse of } \tan\theta}\left(\frac{dy}{dx} \text{ at } t=I\right)$

for $-90° < \theta < 90°$

$\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$

"arctan" also written as "$\tan^{-1}$"

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(6+13t-16t^2)'}{(1+12t)'} = \frac{13-32t}{12}$$

$$\frac{dy}{dx} \text{ at } t = I = \frac{13-32I}{12} \rightarrow \arctan\left(-\left(\frac{13-32I}{12}\right)\right)$$

what is I? Solve $\begin{cases} y=0 \\ t>0 \end{cases} : \begin{cases} 6+13t-16t^2=0 \\ t>0 \end{cases}$

$$\begin{cases} \overbrace{-16t^2}^{a} + \overbrace{13t}^{b} + \overbrace{6}^{c} = 0 \\ t>0 \end{cases} \qquad t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\begin{cases} I = \frac{-13 - \sqrt{13^2-4(-16)(6)}}{2(-16)} = 1.1411\ldots \\ I \neq \frac{-13+\sqrt{553}}{-32} = -0.3286\ldots \end{cases}$$

$I > 0$

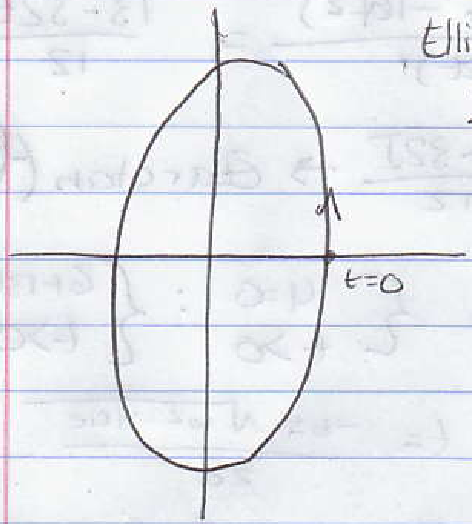
$$I \neq \frac{-13+\sqrt{553}}{-32} = -0.3286\ldots$$

$$\theta = \arctan -\left(\frac{13-32\left(\frac{-13-\sqrt{553}}{-32}\right)}{12}\right) =$$

$$\arctan\left(\frac{\sqrt{553}}{12}\right) = 1.0989 \text{ (radians)}\ldots$$

$$= (1.0989\ldots)\left(\frac{180°}{\pi}\right) = 62.965°\ldots$$

$180° = \pi$, so $\frac{180°}{\pi} = 1$



Ellipse

$$\begin{cases} x = \cos(t) \\ y = 3(\sin(t)) \end{cases}$$

Find $\frac{dy}{dx}$ when $t = \frac{\pi}{4}$

$t = \frac{\pi}{4}$

slope $= \frac{dx}{dy}$ at $t = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(3\sin(t))'}{(\cos(t))'} = \frac{3\cos(t)}{-\sin(t)}$$

$\cos$ at $\pi/4 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \sqrt{2/4}$

At $\pi/4$: $\frac{dy}{dx} = \frac{3(1/\sqrt{2})}{-(1/\sqrt{2})}$

$\boxed{\text{slope} = -3}$