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9/28/10

Today: • Higher Derivatives (2.7)
• Practice

$f' = dy/dx \quad f'' = d^2y/dx^2$

$f^{(3)} = d^3y/dx^3$

$f(x) = 2x^3 + x^2 - 5x + 1$

$f'(x) = 6x^2 + 2x - 5$

$f''(x) = 12x + 2$

$f^{(3)}(x) = 12$

$f^{(4)}(x) = 0$

$f^{(5)}(x) = 0$

$y = f(x)$

$dy = f'(x)\,dx = (6x^2 + 2x - 5)dx$

$d^2y = f''(x)(dx)^2 = (12x+2)\,dx^2$

$d^3y = f^{(3)}(x)(dx)^3 = 12\,dx^3$

$d^4y = f^{(4)}(x)(dx)^4 = 0\,dx^4 = 0$

$d^5y = f^{(5)}(x)(dx)^5 = 0\,dx^5 = 0$

Physics Example

Position = x

time = t

instantaneous { velocity = v, acceleration = a }

jerk = j

$v = \dfrac{dx}{dt}$

$a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$

$j = \dfrac{da}{dt} = \dfrac{d^2v}{dt^2} = \dfrac{d^3x}{dt^3}$

Practice

1. $y = \tan(\ln^3 x - \csc\sqrt{x})$; $\dfrac{dy}{dx} = ?$

$\sec^2(\ln^3 x - \csc\sqrt{x})\left((3(\ln x)^2)\left(\frac{1}{x}\right) + \csc\sqrt{x}\cot\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)\right)$

$\sec^2(\ln^3 x - \csc\sqrt{x})\,\underbrace{[\ln^3 x - \csc\sqrt{x}]'}$

$\to \underbrace{(\ln^3 x)'}_{} - \underbrace{(\csc\sqrt{x})'}_{}$

$\underbrace{((\ln x)^3)'}_{} \qquad (-\csc\sqrt{x}\cot\sqrt{x})\underbrace{(\sqrt{x})'}_{\frac{1}{2\sqrt{x}}}$

$3(\ln x)^2\underbrace{(\ln x)'}_{\frac{1}{x}}$

2. $y = 2\cos^5(x)\, 4^{x+3}$; $\frac{dy}{dx} = ?$ $u = \cos x$

$y = u^5 = \cos^5 x$

$$\frac{dy}{dx} = (2\cos^5 x)\underbrace{(4^{x+3}\ln(4))}_{\substack{(4^{x+3})' \\ (4^{x+3})(\ln 4)\underbrace{(x+3)'}_{1}}} + 4^{x+3}\underbrace{(10(\cos x)^4(-\sin x))}_{\substack{(2\cos^5(x))' \\ 2\cdot 5(\cos^4 x)(-\sin x)}}$$

$$(\cos^5 x)' = \frac{dy}{dx} = \underbrace{\frac{dy}{du}}_{\substack{(u^5)' \\ 5u^4}}\underbrace{\frac{du}{dx}}_{(\cos x)'}$$

$$(4^{3x^2+3})' = 4^{3x^2+3}(\ln 4)(6x)$$

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3. $y = \dfrac{3\sec(2x)}{x^2 + \log_7 \cot x}$; $\frac{dy}{dx} = ?$

$$y' = \frac{[3\sec(2x)\tan(2x)(2)][x^2 + \log_7 \cot x] - [3\sec(2x)]\left[2x\frac{1}{\cot x \ln 7}(-(\csc x)^2)\right]}{(x^2 + \log_7 \cot x)^2}$$

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9. $y = e^{e^x} = e^{(e^x)}$

$u = e^x$ $y = e^u$

$$\frac{dy}{dx} = \underbrace{\frac{dy}{du}}_{\substack{(e^u)' \\ e^u}}\underbrace{\frac{du}{dx}}_{\substack{(e^x)' \\ e^x}} = e^u e^x = e^{e^x} e^x$$

$$\frac{d^2y}{dx^2} = (e^{e^x} e^x)' = (e^{e^x})' e^x + e^{e^x}(e^x)'$$

$= e^{e^x} e^x e^x + e^{e^x} e^x$ ✓

$= e^{e^x} e^x (e^x + 1)$ ✓

$= e^{(e^x + 1)}(e^x + 1)$ ✓

Cont. 9/28/10

7. $y = \tan(\ln(\sin(\cos(x))))$; $\frac{dy}{dx}$

$$\sec^2(\ln(\sin(\cos(x))))\frac{1}{(\sin(\cos(x)))}\left(\cos(\cos x)\right)(-\sin(x))$$

$du/dx = -\sin x$	$u = \cos x$
$dv/du = \cos u$	$v = \sin u$
$dw/dv = 1/v$	$w = \ln v$
$dy/dw = \sec^2 w$	$y = \tan w$

$$\frac{dy}{dx} = \underbrace{\frac{dy}{dw}}_{\sec^2 w}\,\underbrace{\frac{dw}{dv}}_{\frac{1}{v}}\,\underbrace{\frac{dv}{du}}_{\cos u}\,\underbrace{\frac{du}{dx}}_{-\sin x}$$