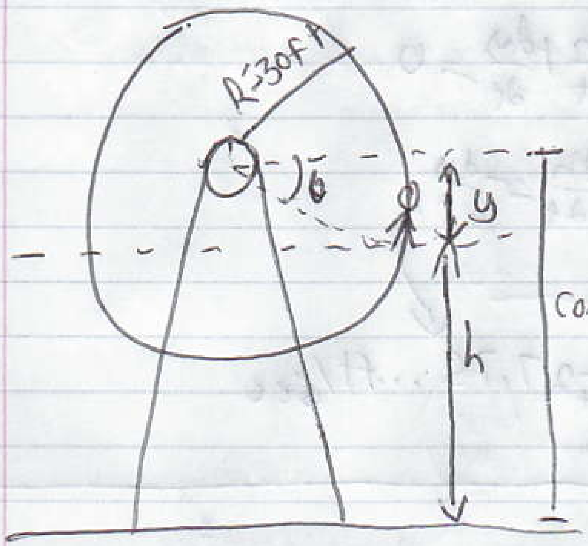
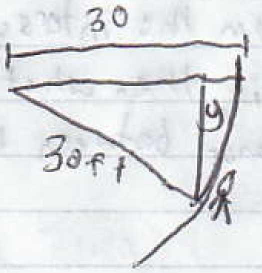
A ferris wheel with a radius of 30ft is spinning at a rate of 0.17 revolution/second. At the moment a rider is 15ft lower than the center of the wheel, what is the rate of change of her height from the ground.

When $y = 15ft$,

$\frac{dh}{dt} = ?$





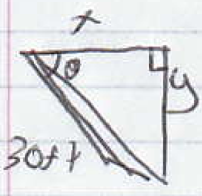
$$\boxed{(\cos\theta)\frac{d\theta}{dt} = \frac{1}{30ft}\frac{dy}{dt}}$$

$$\sin\theta = \frac{y}{30ft} \rightarrow d(\sin\theta) = d\left(\frac{y}{30ft}\right) \rightarrow \frac{\cos\theta\, d\theta}{dt} = \frac{d\left(\frac{y}{30ft}\right)}{dt}$$

$$h + y = c \rightarrow \frac{dh + dy}{dt} = \frac{dc}{dt} \Rightarrow \boxed{\frac{dh}{dt} + \frac{dy}{dt} = 0}$$

constant

Plug in numbers for our moment in time when $y = 15ft$ $\left|\frac{d\theta}{dt}\right| = 2\pi \cdot 0.17/s$



$$\boxed{\cos\theta = \frac{x}{30ft} = \frac{\sqrt{675}\,ft}{30ft} = \frac{\sqrt{675}}{30}}$$

$x = \sqrt{675}\ ft$

$x^2 + y^2 = (30ft)^2$ $\qquad x^2 = 675ft^2$

$x^2 + (15ft)^2 = 30ft^2 \rightarrow x^2 + 225ft^2 = 900ft^2$

Aside:

$$\boxed{\left(\frac{f(x)}{30}\right)' = \left(\frac{1}{30}f(x)\right)' = \frac{1}{30}f'(x)}$$

$$\cos\theta \frac{d\theta}{dt} = \frac{1}{30ft}\frac{dy}{dt}$$

$$\frac{\sqrt{675}}{30}\cdot\left(\frac{0.34\pi}{\text{second}}\right) = \frac{1}{\cancel{30}ft}\frac{dy}{dt}$$

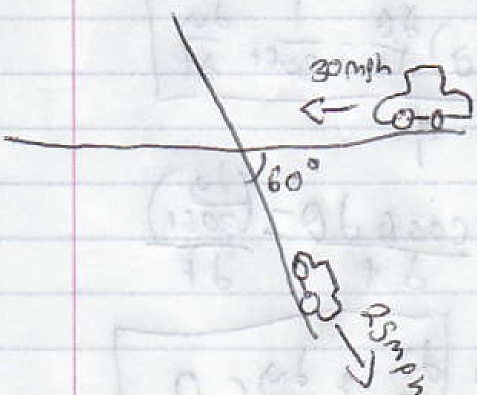
$$\pm\sqrt{675}\cdot 0.34\pi \text{ ft/sec} = \frac{dy}{dt}$$

$$\pm 27.75\ldots \text{ ft/sec} = dy/dt$$

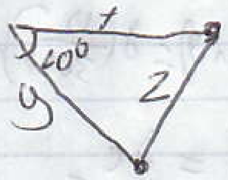
$$\frac{dh}{dt}+\frac{dy}{dt}=0$$

$$\frac{dh}{dt}=-\frac{dy}{dt}$$

$$\rightarrow \frac{dh}{dt} = \mp 27.75\ldots \text{ ft/sec}$$



If the faster car is 4 miles from the intersection & the slower car is 5 miles from it, then what is the rate of change of the distance between the cars?



$$\frac{dx}{dt} = -30 \text{ mph}$$

$$\frac{dy}{dt} = +25 \text{ mph}$$

$$\frac{dz}{dt} = ? \text{ when } \begin{cases} x = 4 \text{ miles} \\ y = 5 \text{ miles} \end{cases}$$

$$z^2 = x^2+y^2-2xy\underbrace{\overbrace{\cos 60^\circ}^{\pi/3}}_{1/2} \quad \text{(Law of Cosines)}$$

$$z^2 = x^2+y^2-xy$$

$$d(z^2) = d(x^2 + y^2 - xy)$$

$$\frac{2z\,dz}{dt} = \frac{2x\,dx + 2y\,dy - ((dx)y + x\,dy)}{dt}$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} - \left(\frac{dx}{dt}y + x\frac{dy}{dt}\right)$$

Now plug in numbers when $x = 4\,mi$, $y = 5\,mi$

$$z^2 = (4^2 + 5^2 - 4\cdot 5)\,mi^2$$

$$z = \sqrt{21}\,mi$$

$$2(\sqrt{21}\,mi)\frac{dz}{dt} = \overbrace{2(4mi)(-30mph)}^{-240} + \overbrace{2(5mi)(25mph)}^{250}$$

$$\underbrace{- \left(\underbrace{(-30mph)(5mi)}_{-150} + \underbrace{(4mi)(25mph)}_{100}\right)}_{-50}$$

$$2(\sqrt{21}\,mi)\frac{dz}{dt} = (-240 + 250 - (-50))\,mi\cdot mph = 60\,mi\cdot mph$$

$$\frac{dz}{dt} = \frac{60}{2\sqrt{21}}\,mph$$

$$= 6{,}5465$$

A rocket in space is firing so that its momentum, which is it's mass times its velocity, is increasing at a rate of $8.0 \cdot 10^4$ kg·m/s², As it burns its fuel, the rocket's mass is decreasing at 43 kg/s

If the rocket's mass is currently $7.2 \cdot 10^4$ kg & its velocity is currently $1.6 \cdot 10^3$ m/s, then what is the curent acceleration?

p = momentum $\quad p = mv \quad d(p) = d(mv)$

m = mass

v = velocity

$$\frac{dp}{dt} = \frac{(dm)v + m\,dv}{dt}$$

$$\frac{dp}{dt} = \frac{dm}{dt}v + m\left(\frac{dv}{dt}\right)$$

acceleration

$\frac{dv}{dt} = ?$ when $m = 7.2 \cdot 10^4$ kg

$v = 1.6 \cdot 10^3$ m/s

$\frac{dp}{dt} = 8.0 \cdot 10^4$ kg m/s²

$\frac{dm}{dt} = -43$ kg/s

$$8.0 \cdot 10^4 \frac{kg \cdot m}{s^2} = \left(-43 \frac{kg}{s}\right)\left(1.6 \cdot 10^3 \frac{m}{s}\right) + \left(7.2 \cdot 10^4\, kg\right)\frac{dv}{dt}$$

$$8.0 \cdot 10^4 \frac{m}{s^2} = -6.88 \cdot 10^4 \frac{m}{s^2} + 7.2 \cdot 10^4 \frac{dv}{dt}$$

$$14.88\,\cancel{\cdot 10^4} \frac{m}{s^2} = 7.2\,\cancel{\cdot 10^4} \frac{dv}{dt}$$

$$2.0666\ldots \frac{m}{s^2} = \frac{dv}{dt}$$