Continuity 3.4

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2:33 AM

F(x) is continues at C" meaning $x \approx c \Rightarrow f(x) \approx f(c)$

Equivalent definitions
1) $\lim_{x \to c} f(x)=f(c)$
2) $0 \neq e$ but $\approx 0 \Rightarrow St(f(c+e))=f(c)$
3) $0 \neq \Delta X$ but $\approx 0 \Rightarrow st(f(c+\Delta X))=f(c)$
4) $0 \neq e \approx 0 \Rightarrow f(c+e) \approx f(c)$

E.G. $F(t) = t^2$ is continuous at 5
2) $0 \neq e \approx 0 \Rightarrow st((5+e)^2) = (5+0)^2 = 5^2 = f(5)$

There are 3 ways f(x) can fail to be continuous at c:
1) f(c*) is Undefined DNE
2) $\lim_{x \to c} f(x)$ DNE
3) $\lim_{x \to c} f(x) \neq f(c)$

$\lim_{x \to 1} f(x) = st[(1+e)^2-1]/(1+e-1) = st\ (2e+e)^2/e$
$= st(2+e) = 2+0 = 2$
But $f(1) = (1^2-1)/1-1 = 0/0$

1) $F(x) = (x^2-1)/(x-1)$
2) $F(x) = 1/(x-3)$

3) EX
$F(x) = X+2 \ : x \neq 4$
$\quad\ \ 5 \quad : x = 4$
$\lim_{x \to 4} f(x) = st(f(4+e)) = st(4+e+2) = 4+0+2 = 6 \neq 5 = f(4)$

Continues Continuous functions are nice for limits
$\lim_{x \to c} f(x) = f(c)$

So which functions are continuous and which x-values?

Fact: If f'(c) exists, then f is continuous at c. Why?
Let $0 \neq \Delta X \approx 0$
By definition….
$St\ [(f(c+\Delta X)-f(c)]/\Delta X = f'(c)$

$\underbrace{St(\Delta X)}_{0}st(f(c+\Delta X)-f(c)/\Delta X = f'(c)\ \underbrace{st\ (\Delta X)}_{0}$

$St(\cancel{\Delta X} * f(x+\Delta X)-f(c)/\cancel{\Delta X} = 0$
$St(f(c+\Delta X)-f(c) = 0$
$St(f(c+\Delta X) - St(f(c)) = 0$
$St(f(c)+\Delta X) - f(c) = 0$
$St(f(c)+\Delta X = f(c)$
$\lim_{x \to c} f(x) = f(c)$

$\lim_{x \to \pi/3} \sin(x) = \sin(\pi/3) = \sqrt{3}/2$

Sine is differentiable at π/3 (and everywhere else), so sine is continuous at π/3 (and everywhere else).

$\lim_{x \to 3} 1/x-3$ DNE
$(1/x-3)' = -1/(x-3)^2$ Not defined at x=3

$\lim_{x \to 5} 1/x-3 = 1/5-3 = 1/5-3 = 1/2$
$(1/x-3)'$ exists at 5 = x, so 1/x-3 is continuous there

√X is not continuous at 0 because $\lim_{x \to 0} \sqrt{X}$ DNE
$\sqrt{0+e}$ is not defined for $0>e\approx 0$, so $st(\sqrt{x+e})$ is not defined so $\lim_{x \to 0} \sqrt{X}$ DNE

Useful rule for limits:
If f is continuous, at L and $\lim_{x \to c} g(x) = L$ then $\lim_{x \to c} f(g(x)) = f(\lim_{x \to c} g(x)) = f(L)$
Same for x->c+ and x->c-

$\lim_{x \to 0} \cos(1/\ln x) = \cos (\lim_{x \to 0} 1/\ln x)$

Cos exists everywhere so Cos is continuous everywhere.

Assuming $\lim_{x \to 0+} 1/\ln x$ Exists
$1/\ln 0.01 = -0.217….$
$1/\ln 0.00001 = 0.108$

One Sided Continuity
Continuity on closed intervals

F is continuous at C is and only if x is infinitely close to C

$X \approx C \Rightarrow F(x) \approx f(c) \Leftrightarrow \text{Lim}_{(x \to c)} f(x) = f(c)$

F is continuous from the right at c $\Leftrightarrow \lim_{(x \to c)+} f(x) = f(c)$

F is continuous from the left at c $\Leftrightarrow c \geq x \approx f(x) \approx f(c) \Leftrightarrow \lim_{(x \to c)-} f(x) = f(c)$

0<e Eapprox0

$\sqrt{6-3x}$ is continuous from the left at 2

Why? Let $f(x) = \sqrt{6-3x}$. Check that $\lim_{(x \to 2)-} f(x) = f(2)$

Let epsilon be a positive infinitesimal, then

$\text{Lim}_{(x \to 2)-} f(x) = st(f(2-e))$ & $f(2) = 0$

IF the limit exists

Check that $st(f(2-e)) = 0$ for all positive infinitesimals e:

$st(f(2-e)) = st(\sqrt{6-3(2-e)}) = st(\sqrt{6-6+e}) = st(\sqrt{e}) = \sqrt{st(e)}$

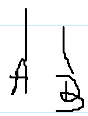
$= \sqrt{0} = 0$

Continuous From left

continuous From right

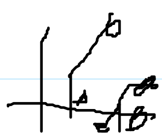
F is continuous on {a,b}

Meaning f is continuous at c for all x in (a,b) and f is continuous from the right at A and f is continuous from the left at B.



continuous from both sides in the middle

Intuitively, this means we can draw the curve y=fx from x=a to x=b without lifting our pencil,marker,pen,ect. From the paper



Not continuous

$f(x) = \sqrt{6-3x}$.

This is continuous on {1,2} because $\lim_{(x \to 2)-} f(x) = f(c)$

Two Sided Continuity

Check

If $c<2$ & $0 \neq 3 = e \approx 0$ then

$St(f(c+e)) = st(\sqrt{6-3(c+e)}) = \sqrt{st(6-3(c+e))} = \sqrt{6-3(c+0)}$

$= f(c) = \sqrt{6-3c}$

So $\lim_{(x \to c)} f(x) = f(c)$

When $c<2$

For all $a<2$, f(x) is continuous on [a,2]

If g(x) is built from

+, *, -, /, e^x, lnx, sinx, cosx, x^n, $\sqrt[n]{x}$ then g(x) is continuous on a closed interval from [a,b] if for all c in [a,b],

g(c) does not involve ln(0), ln(negative), or $\sqrt[\text{even}]{\text{negative}}$.

Watch out for hidden divisions and hidden logarithms.

Tanx=sinx/cosx cotx=cosx/sinx sexc=1/cosx csc = 1/sinx

$\text{Log}_b a = \ln a / \ln b$ $\quad x^y = e^{(\ln x)y}$

If not rational.

e.g. $2^\pi = e^{(\ln 2)\pi}$

$(-2)^\pi = e^{(\ln -2)\pi}$

DNE!

$-2^{(3/5)} = \sqrt[5]{(-2)^3} = \sqrt[5]{-8}$

$\sqrt[4]{\cos(2x)}$ is continuous on $[-\pi/4, \pi/4]$

$-\pi/4 \leq x \leq \pi/4 \Rightarrow -\pi/2 \leq 2x \leq \pi/2 \Rightarrow \cos 2x \geq 0$

$y = \cos\Theta$ we avoid $\sqrt[4]{\text{negative}}$

1/5-x is not continuous on [1,5]: x=5: 1/5-5 = 1/0 is undefined and 5 is in [1,5]

$1 \leq 5 \leq 5$

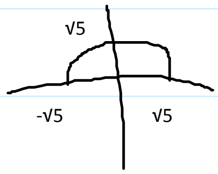
1<=5<=5

$\mathrm{Ln}(x^{2+1})$ is continuous on every [a,b]

Is $\sqrt{}(5-x^2)$ continuous on [-2,2]?

$5-x^2 \geq 0 \leq> 5 \geq x^2 = \backslash x \backslash^2$

$2=\sqrt{}(4)$, so $-2 \leq x \leq 2$ <=> $-\sqrt{4} \leq x \leq \sqrt{4}$ => $-\sqrt{5} \leq x \leq \sqrt{5}$



Actually, $\sqrt{5}-x^2$ is continuous on the bigger interval $[-\sqrt{5}, \sqrt{5}]$