10/14/10

More properties of continuous functions (3.8)

last time: IVT

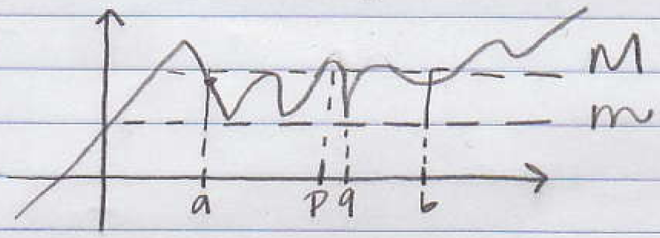
EVT

RT

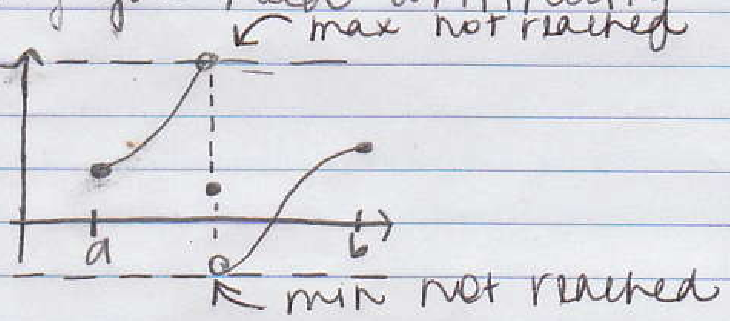
MVT

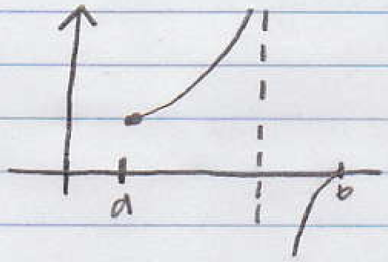
Extreme value theorem (EVT)

If f is a continuous on $[a,b]$, then f has maximum value M at some p in $[a,b]$ & a minimum value at some q in $[a,b]$ (these max & min are only over $[a,b]$)



Why you need continuity:





no min or max; no bounds at all

anna garza

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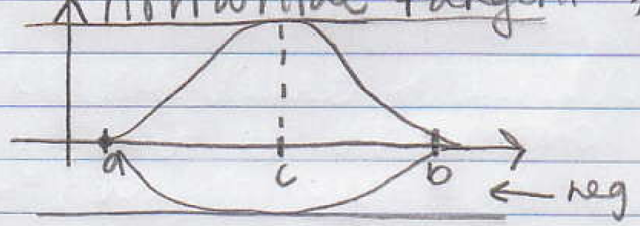
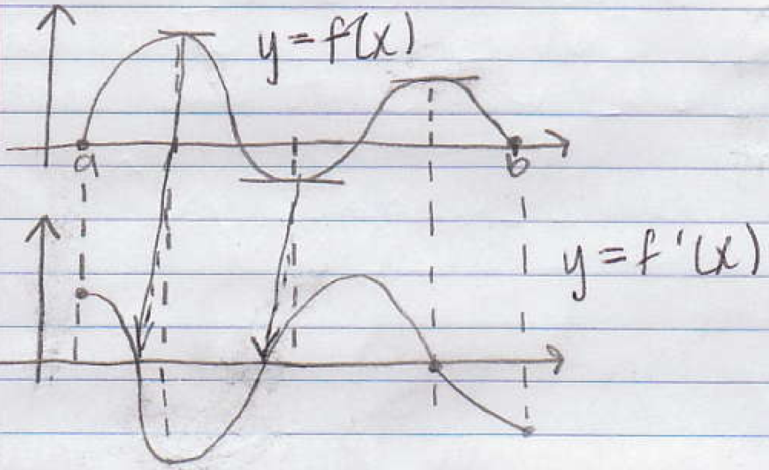
Rolle's Theorem

If f is cts on $[a,b]$, $f'(c)$ exists for all c in (a,b), and $f(a) = f(b) = 0$, then $f'(c) = 0$ for some c in (a,b)

horizontal tangent $f'(x)=0$



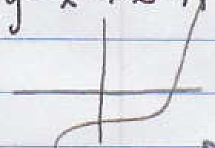


Last time, we proved that $x^5+x-14=0$ for some x in $(1,2)$

Now, we can prove that $x^5+x-14=0$ does not have more than one (real) solution

If $a<b$ & $a^5+a-14 = b^5+b-14 = 0$, then, since $f(x) = x^5+x-14$, is cts everywhere & differentiable everywhere — $f'(x) = 5x^4+1$ —, Roelle's theorem says the $f'(c)=0$ for some c in (a,b)

y=x^5+x-14



But $f'(x) = \underbrace{5x^4}_{\underbrace{5(x^2)^2}_{\geq 0}}+1 \geq 5 \cdot 0 + 1 \geq$ so $f'(c)=0$ is impossible

so $f(x)=0$ can't have 2 real solutions

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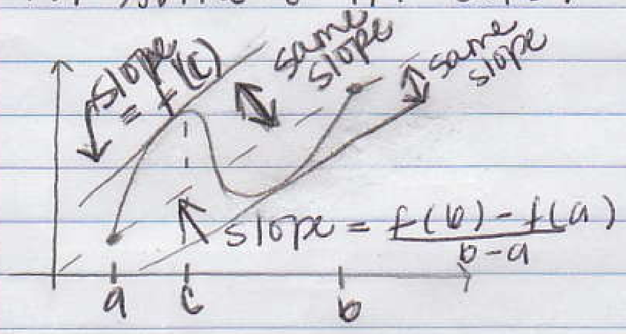
Mean Value Theorem

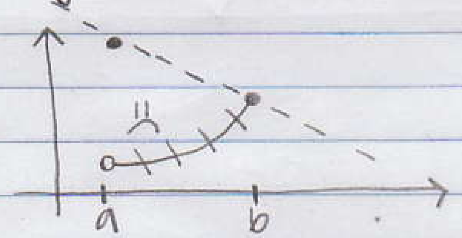
If f is cts. on $[a,b]$ and $f'(c)$ exists for all c in (a,b),

then $\underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{avg. rate of change}} = \underbrace{f'(c)}_{\text{some instaneous rate of change}}$

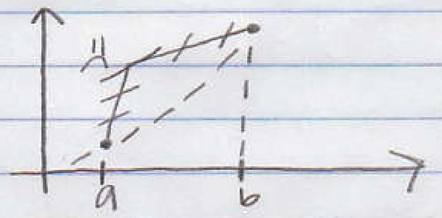
for some c in (a,b)



Why MVT needs continuity on $[a,b]$



Why MVT needs differentiality on (a,b)



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Example: If a tank of water had 15 gallons of water in it, and 8 min later, it has 3 gallons, then at some moment in time, the instantaneous rate of leaking was $\frac{15-3}{8} = 1.5$ gallons/min