Review Notes Day Three

The average value of $f(x)$ from $x=a$ to $x=b$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$

L'Hôpital's Rule

$$\lim_{x\to a} \frac{f(x) \nearrow 0}{g(x) \searrow 0} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \neq \lim_{x\to a} \left(\frac{f(x)}{g(x)}\right)'$$

$$\lim_{x\to a} \frac{f(x) \nearrow \pm\infty}{g(x) \searrow \pm\infty} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \neq \lim_{x\to a} \left(\frac{f(x)}{g(x)}\right)'$$

$$\lim_{x\to 1^+} \frac{\sin(\pi x)}{\sqrt{x-1}} = \left\{\begin{array}{l} x = 1+\varepsilon \\ 0 < \varepsilon \approx 0 \end{array}\right\} \Rightarrow \sin(\pi x) \overset{\text{sin is cont.}}{\approx} \sin(\pi \cdot 1) = \sin\pi = 0$$

$$\sqrt{x-1} = \sqrt{1+\varepsilon-1} = \sqrt{\varepsilon} = 0$$

$$\lim_{x\to 1^+} \frac{(\sin \pi x)'}{\left((x-1)^{\frac{1}{2}}\right)'} = \lim_{x\to 1^+} \frac{(\cos \pi x)(\pi x)'}{\frac{1}{2}(x-1)^{\frac{1}{2}-1}(x-1)'}$$

$$= \lim_{x \to 1^+} \frac{(\cos \pi x)\,\pi}{\frac{1}{2}(x-1)^{-1/2}(1-0)}$$

$$= \lim_{x \to 1^+} \frac{2\pi \cos \pi x}{1/\sqrt{x-1}}$$

$$= \lim_{x \to 1^+} 2\pi \underbrace{(\cos \pi x)}_{\text{cos is cont.}} \underbrace{\sqrt{x-1}}_{0}$$

$$x = 1+\varepsilon \Rightarrow \cos(\pi x) \approx \cos(\pi \cdot 1) = \cos \pi = -1 \quad = 2\pi(-1)(0) = 0$$

$$\lim_{x \to \infty} (\overset{\text{grows much faster}}{x^2} - \underbrace{x \ln x}_{\infty})$$

$x^2 \to \infty$

Dealing with "$\infty - \infty$"

Factor out ∞,

actually factor out the "biggest" ∞

Let H be positive infinite

Plug in $x = H$

Guess which one is bigger.

$$H^2 - H\ln H = \underbrace{H^2}_{\text{inf.}} \overbrace{\Big(\underbrace{1 - \frac{\ln H}{H}}_{0}\Big)}^{?} \approx 1 \to \text{inf positive}$$

~~or~~ or $H \ln H \left(\frac{H}{\ln H} - 1\right) \to$ inf. positive

$\lim_{x\to\infty} \frac{x}{\ln x}$ $\lim_{x\to\infty} \ln x$

$$\lim_{x\to\infty} \frac{\ln x \nearrow \infty}{x \searrow \infty} = \lim_{x\to\infty} \frac{1/x}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{x \nearrow \infty}{\ln x \, \infty} = \lim_{x\to\infty} \frac{1}{1/x} = \lim_{x\to\infty} x = \infty$$

Both approaches yield $\lim_{x\to\infty} (x^2 - x\ln x) = \infty$

Another form of $\infty - \infty$

$$\lim_{x\to 0} \frac{1}{\underbrace{x}_{\infty}} - \frac{1}{\underbrace{\sqrt{x}}_{\infty}}$$

$$\underbrace{\frac{1}{x}}_{\infty} (1 - \sqrt{x}) = \infty$$

$$(1 - 0) = 1$$

$$\underbrace{\frac{1}{\varepsilon}}_{\text{positive infinite}} \underbrace{(1 - \sqrt{\varepsilon})}_{\approx 1} \text{ is positive infinite}$$

Combine fractions if you have an $\infty - \infty$ (or factor out one fraction)

Another trick

$$(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = a-b$$

$$\lim_{x\to 2} \frac{3x-6}{\sqrt{x+7}-\underbrace{3}_{\sqrt{9}}} = \lim_{x\to 2} \frac{(3x+6)(\sqrt{x+7}+\sqrt{9})}{(\sqrt{x+7}-\sqrt{9})(\sqrt{x+7}+\sqrt{9})}$$

$$= \lim_{x\to 2} \frac{(3x+6)(\sqrt{x+7}+3)}{x+7-9}$$

Factor!!

$$\lim_{x\to 2} \frac{3(x-2)(\sqrt{x+7}+3)}{x-2} =$$

$$\lim_{x\to 2} 3(\sqrt{x+7}+3) = 3(\sqrt{2+7}+3) =$$

$$3(3+3) = \boxed{18}$$

Abbreviations

"big" for infinite hyperreal

"medium" for finite non-infinitesimal

"small" for infinitesimal

$(+\text{big}) + (+\text{big}) = +\text{big}$

$(+\text{big}) - (+\text{big}) = ?$

$(-\text{big}) + (+\text{big}) = -\text{big}$

$(+\text{big})(-\text{medium}) = -\text{big}$

$(+\text{big})(+\text{small}) = ?$

$$(+\text{small})(-\text{big}) = ?$$

$$\lim_{x \to 0^+} \underbrace{x}_{0} \underbrace{\ln x}_{-\text{inf}}$$

$$\lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{1/x}{-1/x^2} =$$

$$= \lim_{x \to 0^+} \frac{(1/x)(x^2)}{-1/x^2\,(x^2)} = \lim_{x \to 0^+} \frac{x}{-1} = \frac{0}{-1} = 0$$

$$\frac{\text{small}}{\text{big}} = \text{small}$$

$$\text{small} \pm \text{small} = \text{small}$$

$$\frac{\text{medium}}{\text{small}} = \text{big} \qquad \lim_{x \to 3^-} \frac{x^2 - 5}{2x - 6}$$

$x^2-5 \to 9 - 5 = 4$

$2x-6 \to 0$ Plug in $3 - \varepsilon$

$$\frac{+\text{ medium}}{-\text{small}} = -\text{big} \quad \begin{cases} (3-\varepsilon)^2 - 5 \approx 4 \\ 2(3-\varepsilon) - 6 = 6 - 6 - 2\varepsilon = -2 \end{cases}$$

is small $-$

AH

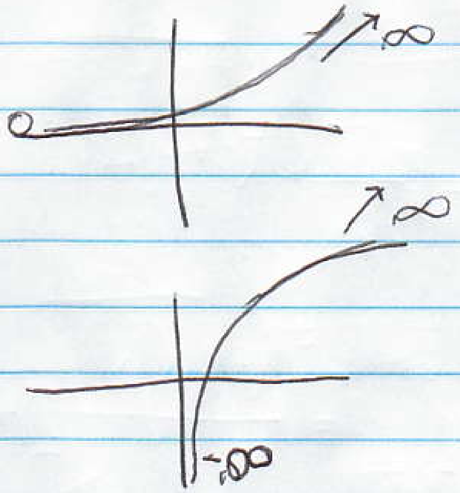
$-$ infinity

$e^{+big} = +big$

$e^{-big} = +smal$

$\ln(+big) = +big$

$\ln(+small) = +big$



Review Session 11 AM – 1 PM
Cowart 211
Saturday