03/09/2010

Cal II

Last:

(9.5) Linear Diff. Equation's

$y'(x) + p'(x)\, y(x) = q(x)$

To Find $y(x)$:

Substitute $u(x) = e^{P(x)}\, y(x)$

$u'(x) = e^{P(x)}\, q(x)$

integrate

$u(x) = \int e^{P(x)} q(x)\, dx$

<u>Example</u> 1

$$xy' + y = 3x^2$$

now re-write $\rightarrow$ $y' + \frac{y}{x} = 3x$

$$\underbrace{y' + \frac{1}{x}}_{p'} = \underbrace{3x}_{q}$$

so if $p' = \frac{1}{x} \Leftrightarrow p = \ln|x|$

two cases $\begin{cases} 1)\ x > 0 \Rightarrow p = \ln x \\ 2)\ x < 0 \Rightarrow p = \ln(-x) \end{cases}$

First case: $u > 0$

$e^{P} = e^{\ln x} = x$

- $u = e^{P} y = xy$
- $u' = xy' + x'y = xy' + 1y = xy' + y = 3x^2 = x \cdot 3x = \overbrace{x}^{e^P} \cdot \overbrace{3x}^{q} = e^{P} q$
- $u' = e^{P} q = x \cdot 3x = 3x^2 \Rightarrow u = \int 3x^2\, dx = x^2 + C$

CONT....

$$u = xy \Rightarrow y = \frac{u}{x} = \frac{x^2+c}{x} = x^2 + \frac{c}{x}$$

Let's check if it works: $y = x^2 + \frac{c}{x}$

$$y' = 2x - \frac{c}{x^2}$$

$$xy' = \left[2x^2 - \frac{c}{x}\right] + \left[x^2 + \frac{c}{x}\right] = 3x^2$$ ✓ correct

★ When you have $\boxed{y' + p'y = q}$ you have to use $\boxed{u = e^p y = xy}$ to get a much simpler equation.

Example 2

Rule: $y' + p'y = q \Rightarrow$ use $u = e^p y$

$\Downarrow$

$u' = e^p q$

$$x^2y' + x^3y = 3x^3$$

$$y' + \underbrace{x}_{p'}y = \underbrace{3x^1}_{q}$$

$$p' = x \Rightarrow \text{pick } p = \frac{x^2}{2}$$

$$u' = e^p q = e^{x^2/x} \cdot 3x$$

$$u = \int e^{x^2/x}\, 3x\, dx$$

substitution:

- $w = x^2/2$
- $\frac{dw}{dx} = \frac{2x}{2} = x$
- $dw = x\,dx$

now using sub:

$$u = \int e^w \, 3dw$$

$$u = 3\int e^w dw = 3e^w + \underbrace{C} = 3x^{x^2/2} + C$$

important!

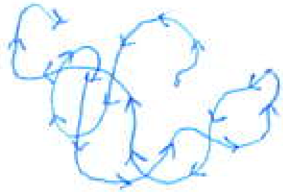
$$u = e^P y \Rightarrow y = \frac{u}{e^P} = \frac{3e^{x^2/2} + C}{e^{x^2/2}} = \boxed{3 + Ce^{-x^2/2}}$$

Let's check: $y = 3 + Ce^{-x^2/2}$

$$y' = 0 + Ce^{-x^2/2} \quad (-x^2/2)'$$
$$y = -Cxe^{-x^2/2}$$
$$x^2 y' = -Cx^3 e^{-x^2/2}$$
$$x^3 y = x^3 (3 + Ce^{-x^2/2}) = 3x^3 + Cx^3 e^{-x^2/2}$$
$$x^2 y' + x^3 y = -Cx^3 e^{-x^2/2} + 3x^3 + Cx^3 e^{-x^2/2} = 3x^3 \checkmark$$ correct.

Parametric Curves 10.1

Imagine an ant crawling along the xy-plane

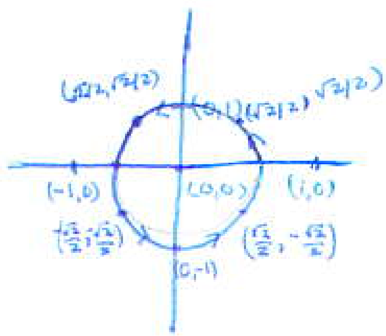


$x = f(t) =$ x-coordinate at time t.

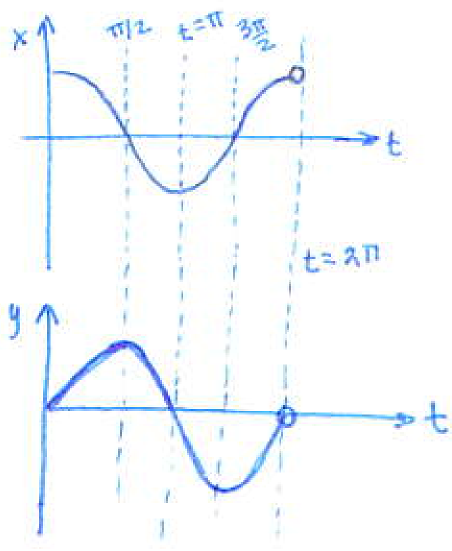
$y = g(t) =$ y-coordinate at time t.

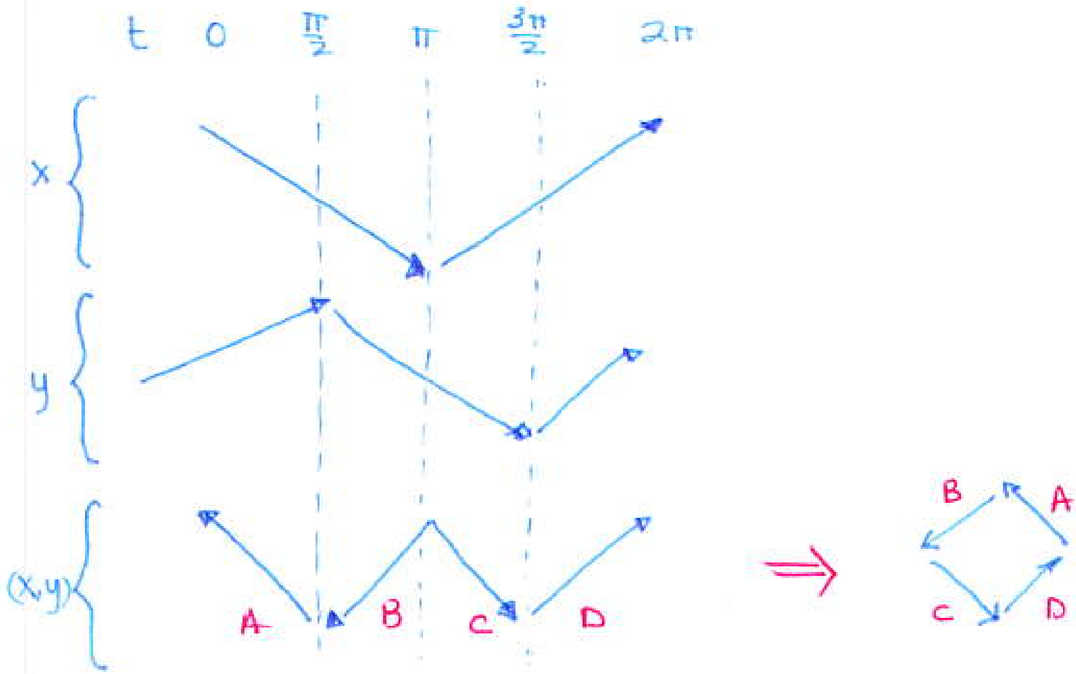
Classic Curve

$$\begin{cases} x = \cos t \\ y = \sin t \\ 0 \le t < 2\pi \end{cases}$$



t	x	y
0	1	0
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/2$	0	1
$\frac{3\pi}{4}$	$-\sqrt{2}/2$	$\sqrt{2}/2$
π	-1	0
$5\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$





t
0
π/2
π
3π/2
2π
x
y
(x,y)
A
B
C
D
B
A
C
D