Last time

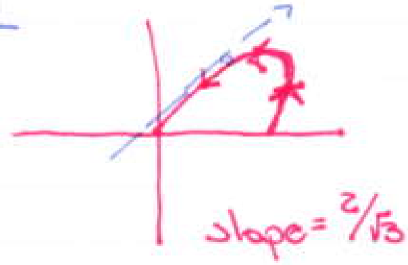
$x = \cos(t)$
$y = \sin(2t)$

Hi you guys
:) tried my best
hope you understand

Look at

(I didn't have liquid paper :( )

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos(2t)}{-\sin t}$$



$$\left.\frac{dy}{dx}\right|_{t=\pi/3} = \text{slope}$$

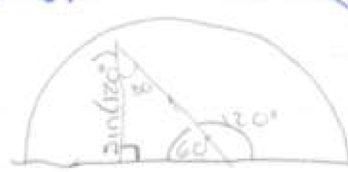
$$\frac{2(-1/2)}{-\sqrt{3}/2} = \frac{2}{\sqrt{3}}$$

What is the equation for the tangent line?

$$(x,y)\Big|_{t=\pi/3} = \left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$

$\cos(\pi/3)$ $\sin(2\pi/3)$

$y - b = m(x-a)$
(point-slope)



$$y - \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}}\left(x - \frac{1}{2}\right)$$

tangent line

Direction of curve	dy/dt: −	0	+
dx/dt: −	↙	← left	↖
0	↓ down		↑ up
+	↘	right	↗

When $\frac{dx}{dt} = \frac{dy}{dt} \Rightarrow$ do more work to understand

Try L'Hospital's Rule

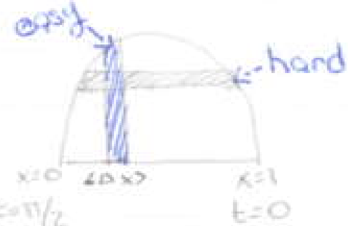
$\frac{dy}{dt} = 0$ and $dx/dt \neq 0$
there is horizontal tangent

when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0 \Rightarrow$ vertical tangent

Now area (again.)

Find area between x-axis and the curve

$\begin{cases} x=\cos(t) \\ y=\sin(2t) \end{cases}$ for $0 \le t \le \pi/2$



$x=\cos(\pi/2)=0$ $\quad$ $x=\cos(0)=1$

$\text{Area}=\sum_{n=1}^{k} y_n^* \Delta x$

$A=\int y\,dx \Rightarrow \int_{x=0}^{x=1} y\, {}^{dy}/_{dt}\cdot dt$

$\Rightarrow \int_{t=\pi/2}^{t=0} (\sin(2t))(-\sin(t)) \Rightarrow \int_0^{\pi/2} \sin(2t)\sin(t)\,dt$

$\Rightarrow \int_0^{\pi/2} 2(\cos(t)\sin(t))\sin t) \Rightarrow 2\int_0^{\pi/2} \sin^2(t)\cos(t)\,dt$

$u=\sin(t)$

$\frac{du}{dt}=\cos(t)$

$du=\cos(t)\,dt$

$=2\int_0^1 u^2\,du \Rightarrow 2\, {}^{u^3}/_{3} \Big|_0^1 \Rightarrow \frac{2(1)^3}{3}-\frac{2(0)^3}{3}=\boxed{2/3}$

$t=0 \rightarrow \sin(0)=0$

$t=\pi/2 \Rightarrow u=\sin\pi/2=1$

In General, area is

$\int y\,dx$ or $\int x\,dy$

NEXT = Arc Lenght: $\int ds$

Find the length of the curve $\begin{cases} x=\cos(t) \\ y=\sin(2t) \end{cases}$ from $t=0$ to $t=\pi/2$

$L=\int_0^{\pi/2} ds = \int_0^{\pi/2}\sqrt{1+({}^{dy}/_{dx})^2}\,dx = \int_0^{\pi/2}\sqrt{1+({}^{dy}/_{\cancel{dx}^2})\cancel{dx^2}} = \int_0^{\pi/2}\sqrt{dx^2+dy^2}$ Next Page

$$L = \int_0^{\pi/2} \sqrt{dx^2 + dy^2} \cdot \frac{dt}{\sqrt{dt^2}} = \int_0^{\pi/2} \sqrt{\left(\frac{dx^2}{dt^2}\right) + \left(\frac{dy}{dt^2}\right)}\, dt$$

$$= \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \checkmark$$

so then $\begin{cases} x = \cos(t) \\ y = \sin(2t) \end{cases}$

$L = \int_0^{\pi/2} \sqrt{(-\sin t)^2 + (2\cos 2t)^2}\, dt$ ← No formula for it. You could use say Simpson's Rule to estimate L

Concavity

concave up (cu)
concave down (cd)

How to determine concavity for parametric curves?

$y = h(x) \begin{cases} cu & \text{if } h''(x) > 0 \\ cd & \text{if } h''(x) < 0 \end{cases}$

Short Answer: $\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}$

At $t = \pi/3$ our curve of $\begin{cases} x = \cos(t) \\ y = \sin(2t) \end{cases}$

is clearly concave down,

the slope we found was $2\cos(2t)/-\sin(t)$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \cdot \frac{dy}{dx} = \frac{dm}{dx} = \frac{dm/dt}{dx/dt} = \frac{\frac{d}{dt}\left(2\cos(2t)/-\sin(t)\right)}{-\sin(t)} = \frac{4\sin(t)\sin 2(t) + 2\cos(t)\cos(2t)}{-\sin^3(t)}$$