Marcelo
Rodriguez

Today: 11.2 } series
Tomorrow: 11.2

Thursday: Review
Monday: MT2
Bring Calculator & 1 sheet of notes

$$\sum_{n=0}^{5} n = 0+1+2+3+4+5 = 15$$

Finite-length series

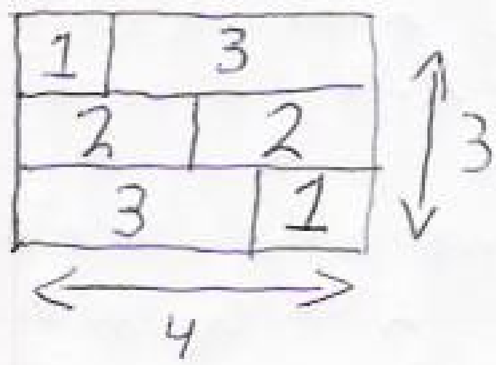
$$\sum_{n=0}^{5} (-1)^n n^2 = (-1)^0 0^2 + (-1)^1 1^2 + (-1)^2 2^2 + (-1)^3 3^2 + (-1)^4 4^2 + (-1)^5 5^2 = 0-1+4-9+16-25 = -15$$

Infinite Series

$$\sum_{k=0}^{\infty} a_k = \lim_{n \to \infty} \sum_{k=0}^{n} a_k$$

$$\sum_{k=0}^{\infty} k = \lim_{n \to \infty} \sum_{k=0}^{n} k = \lim_{n \to \infty} \underbrace{(0+1+2+\dots+n)}_{\frac{n(n+1)}{2}} =$$

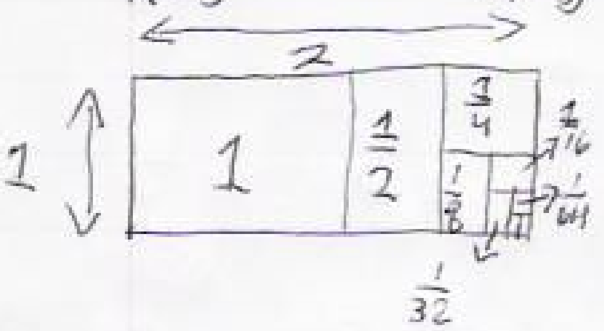
$$\lim_{n \to \infty} \frac{n(n+1)}{2} = \infty \text{ (Divergent)}$$



$$1+2+3=\frac{3\cdot 4}{2}$$

$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

$$\sum_{k=0}^{\infty} 2^{-k}=\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k=2$$



Area$=2$

$1=\left(\frac{1}{2}\right)^0 \quad \frac{1}{2}=\left(\frac{1}{2}\right)^1 \quad \frac{1}{4}=\left(\frac{1}{2}\right)^2 \quad \cdots\cdots$

$$\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\cdots\cdots=2$$

$$\Rightarrow \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{4}+\cdots\frac{1}{2n}\right)=2$$

Recall: $\lim_{n\to\infty} r^n=\begin{cases}0: & -1<r<1\\ 1: & r=1\\ \text{divergent} & \text{otherwise}\end{cases}$

$$\sum_{k=0}^{n}\left(\frac{1}{2}\right)^k=1+\frac{1}{2}+\underbrace{\left(\frac{1}{2}\right)^2}_{\frac{1}{4}}+\underbrace{\left(\frac{1}{2}\right)^3}_{\frac{1}{8}}+\underbrace{\left(\frac{1}{2}\right)^4}_{\frac{1}{16}}+\cdots+\underbrace{\left(\frac{1}{2}\right)^n}_{\frac{1}{2^n}} \implies$$

next page.

$$\left(1+\frac{1}{2}\right)\sum_{k=0}^{n}\left(\frac{1}{2}\right)^k=\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\cdots+\frac{1}{2^n}\right)$$

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n} \text{ or } + \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots+\frac{1}{2^{n+1}}$$

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n} \text{ or } -\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16}\cdots-\frac{1}{2^{n+1}}$$

$$=\left(1-\frac{1}{2}\right)\sum_{k=0}^{n}\left(\frac{1}{2}\right)^k \Rightarrow \frac{1-\frac{1}{2^{n+1}}}{1-\frac{1}{2}}=\sum_{k=0}^{n}\left(\frac{1}{2}\right)^k$$

$$\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k=\lim_{n\to\infty}\frac{1-\frac{1}{2^{n+1}}}{1-\frac{1}{2}}=\frac{1-0}{1-\frac{1}{2}}=\frac{1}{\frac{1}{2}}=2=1-\frac{1}{2^{n+1}}$$

If $-1<r<1$ then

$$\sum_{k=0}^{n} r^k = \cancel{\lim}\ \frac{1-r^{n+1}}{1-r}$$

$$\sum_{k=0}^{\infty} r^k = \lim_{n\to\infty}\frac{1-r^{n+1}}{1-r}=\frac{1}{1-r}$$

↑Geometric Series

$$\sum_{k=0}^{\infty} 7\left(-\frac{1}{3}\right)^k = 7\sum_{k=0}^{\infty}\left(-\frac{1}{3}\right)^k = 7\left(\frac{1}{1-\left(-\frac{1}{3}\right)}\right) = 7\frac{1}{4/3}=\frac{21}{4}$$

$$7-\frac{7}{3}+\frac{7}{9}-\frac{7}{27}+\frac{7}{81}-\frac{7}{243}+\frac{7}{729}\cdots=\frac{21}{4}$$

$$\frac{1}{15}+\frac{1}{75}+\frac{1}{375}+\dots=\sum_{k=0}^{\infty}\frac{1}{15\cdot 5^k}=\frac{1}{15}\left(\frac{1}{1-\frac{1}{5}}\right)=\frac{1}{15}\;\frac{1}{4/5}=\frac{1}{12}$$

$$\frac{1}{15}\left(\frac{1}{5}\right)^k$$

$15=15\cdot 5^0$

$75=15\cdot 5^1$

$375=15\cdot 5^2$

$$\sum_{k=3}^{\infty}\frac{(-3)^k}{4^{k+2}}=?=\frac{(-3)^3}{4^{3+2}}+\frac{(-3)^4}{4^{4+2}}+\frac{(-3)^5}{4^{5+2}}$$

$$\sum_{k=3}^{\infty}\frac{(-3)^k}{4^{k+2}}=\sum_{k=3}^{\infty}\left(\frac{-27}{1024}\right)\frac{(-3)^k/-27}{4^{n+2}/1024}=\frac{-27}{1024}\sum_{k=3}^{\infty}\frac{(-3)^{k-3}}{4^{k+2-(3+2)}}$$

$$=\frac{-27}{1024}+\frac{81}{4096}-\frac{243}{16384}$$

$$\sum_{k=3}^{\infty}\frac{(-3)^k}{4^{k+2}}=\frac{-27}{1024}\sum_{k=3}^{\infty}\left(\frac{-3}{4}\right)^{k-3}=\frac{-27}{1024}\left(\left(\frac{-3}{4}\right)^{3-3}+\left(\frac{-3}{4}\right)^{4-3}+\left(\frac{-3}{4}\right)^{5-3}\dots\right.$$

$$=\frac{-27}{1024}\left(\left(\frac{-3}{4}\right)^0+\left(\frac{-3}{4}\right)^1+\left(\frac{-3}{4}\right)^2+\left(\frac{-3}{4}\right)^3+\dots\right.$$

$$=\frac{-27}{1024}\sum_{k=0}^{\infty}\left(-\frac{3}{4}\right)^k=\frac{-27}{1024}\cdot\frac{1}{1-\left(-\frac{3}{4}\right)}$$

$$=\frac{-27}{1024}\cdot\frac{1}{7/4}=\frac{-27\cdot 4}{1024\cdot 7}=\frac{-108}{7168}=\frac{-27}{1792}$$

A telescoping sum

$$\sum_{k=1}^{\infty}\frac{1}{k(k+1)}=?$$

Partial Fractions $\frac{1}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1}$

$$\underbrace{1 = A(k+1) + Bk}_{\text{For all real numbers}} \begin{cases} k=-1 \Rightarrow 1 = 0 + B(-1) \\ k=0 \Rightarrow 1 = A(1) + 0 = A \Rightarrow A = 1 \end{cases}$$

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right) = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots = 1$$

$$\sum_{k=1}^{\infty} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right) = \lim_{n\to\infty} \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} \cdots - \frac{1}{n+1}\right)$$

$$\lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = 1 - 0 = 1$$

Surprising without partial fractions

$$1 = \sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} \cdots$$