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Today : $\begin{cases} 11.5 \text{ (alternating series)} \\ \text{HW 10 due @ 5PM} \end{cases}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = \frac{(-1)^{1+1}}{\sqrt{1}} + \frac{(-1)^{2+1}}{\sqrt{2}} + \frac{(-1)^{3+1}}{\sqrt{3}}$$

$$+ \frac{(-1)^{4+1}}{\sqrt{4}} + \dots = \frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots$$

<u>alternates</u> between positive and negative terms

We'll show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ is convergent

But $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is divergent : It's $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ is $\begin{cases} \text{convergent: } p > 1 \\ \text{divergent: } p \leq 1 \end{cases}$

$\frac{1}{\sqrt{1}} = 1$

$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} \approx 0.292893$

$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \approx 0.870243$

$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} \approx 0.370243$

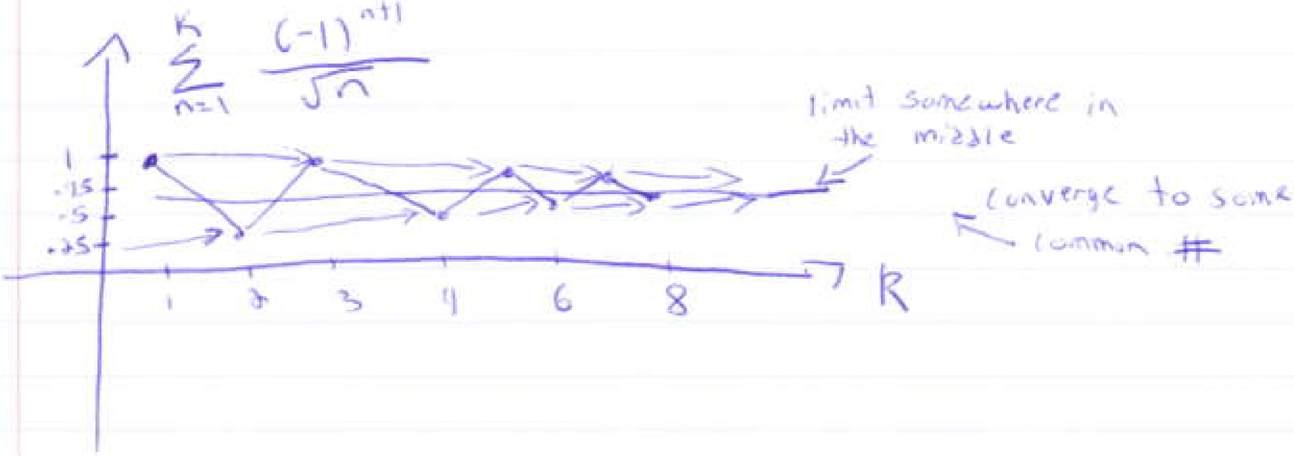
$\frac{1}{\sqrt{1}} - \dots + \frac{1}{\sqrt{5}} \approx 0.817457$

plot

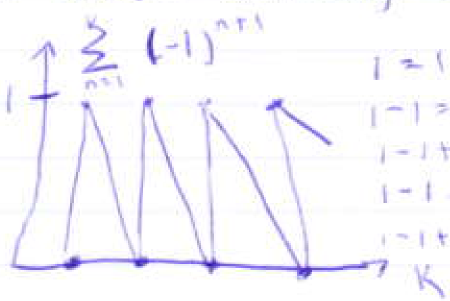
$\frac{1}{\sqrt{1}} - \dots - \frac{1}{\sqrt{6}} \approx 0.408209$

$\frac{1}{\sqrt{1}} - \dots + \frac{1}{\sqrt{7}} \approx 0.787173$

$\frac{1}{\sqrt{1}} - \dots - \frac{1}{\sqrt{8}} \approx 0.43362$



A divergent alternating series : $\sum_{n=1}^{\infty} (-1)^{n+1} = 1-1+1-1+1-1+1-1+1 \dots$



$1 = 1$
$1-1=0$
$1-1+1=1$
$1-1+1-1=0$
$1-1+1-1+1=1$

"Alternating series test"

If $b_1 \geq b_2 \geq b_3 \geq \dots \geq 0$,
and $\lim_{n\to\infty} b_n = 0$,

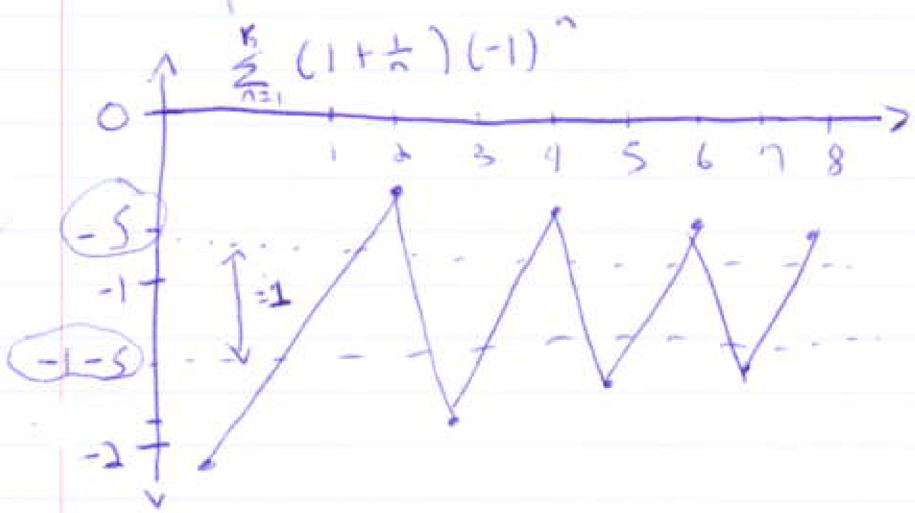
then $b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \dots$
is convergent

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)(-1)^n =$$

$$= \left(1+\frac{1}{1}\right)(-1)^1 + \left(1+\frac{1}{2}\right)(-1)^2 + \left(1+\frac{1}{3}\right)(-1)^3 + \dots$$

$$= -\left(1+\frac{1}{1}\right) + \left(1+\frac{1}{2}\right) - \left(1+\frac{1}{3}\right) + \left(1+\frac{1}{4}\right) - \left(1+\frac{1}{5}\right) + \dots$$

K	$\sum_{n=1}^{K} \left(1+\frac{1}{n}\right)(-1)^n$
1	-2
2	$-2+\frac{3}{2} = -\frac{1}{2} = -.5$
3	$-2+\frac{3}{2}-\frac{4}{3} = \frac{-11}{6} \approx -1.833$
4	$-2+\frac{3}{2}-\frac{4}{3}+\frac{5}{4} = \frac{-7}{12} \approx -.5833$
5	$-2+\frac{3}{2}-\frac{4}{3}+\frac{5}{4}-\frac{6}{5} = \quad \approx -1.7833$
6	$-2+\dots+7/6 \approx -0.61667$
7	$-2+\dots-8/7 \approx -1.75952$
8	$-2+\dots+9/8 \approx -0.63452$



cannot converge to a single #

step sizes = 1

$\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\dots$ is convergent:

$\frac{1}{1} \geq \frac{1}{2} \geq \frac{1}{3} \geq \frac{1}{4} \geq \frac{1}{5} \geq \frac{1}{6} \geq \dots \geq 0$

and $\lim_{n\to\infty} \frac{1}{n} = 0$, so $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \dots$

is convergent by the Alternating Series Test

Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)(-1)^n = -S + \sum_{n=1}^{\infty}(-1)^n \leftarrow \text{divergent}$$

$$-1+1-1+1-1+1-1\ldots$$

$$\underbrace{\sum_{n=1}^{\infty}\frac{(-1)^n}{n}}_{-\frac{1}{1}+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}} = -S$$