The last week:

Review:

Chp. 7 } today (tomorrow?) } $\int_a^b f(x)dx$

Chp. 6, Chp. 8 } Tuesday? } Volumes, Areas, Lengths

Chp. 9, Chp. 10 } Wednesday? } Parametrics, Polar, Diff. equations

Chp. 11 } Thursday (Wednesday too?) } $\sum_{n=0}^{\infty} c_n(x-a)^n$

Substitution! $\boxed{\int_a^b f(g(x))g'(x)dx = \int_c^d f(u)\,du \quad \begin{matrix} u=g(x) & d=g(b) \\ du=g'(x)dx & c=g(a) \end{matrix}}$

$$\boxed{\begin{matrix} \int f'(g(x))g'(x)dx = \int f'(u)du \\ \parallel \\ f(g(x))+c = f(u)+c \end{matrix}}$$

Integration by Parts:

$$\int fg'dx = fg - \int f'g\,dx$$

$$= \int \underbrace{x}_{f}\underbrace{e^{2x}}_{g'}dx \qquad f'=1, \quad g=\frac{1}{2}e^{2x}$$

$$= \underbrace{x\left(\frac{1}{2}e^{2x}\right)}_{fg} - \int \underbrace{1}_{f'}\underbrace{\frac{1}{2}e^{2x}}_{g}dx = \frac{x}{2}e^{2x} - \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) = \left(\frac{x}{2}-\frac{1}{4}\right)e^{2x}+C$$

$$\int \underbrace{(\ln x)}_{f}\underbrace{x}_{g'}dx = \underbrace{(\ln x)\frac{x^2}{2}}_{fg} - \int \underbrace{\overbrace{\left(\frac{1}{x}\right)\frac{x^2}{2}}^{\frac{x}{2}}}_{f'g} = \frac{x^2}{2}\ln x - \frac{x^2}{4}+C$$

$$f'=\frac{1}{x} \quad g=\frac{x^2}{2}$$

$$\int_1^2 \frac{dx}{x\ln x} = \int_{\ln 1}^{\ln 2} du\, \underbrace{\frac{1}{u}}_{\frac{1}{\ln x}} = \ln|u|\Big|_{\ln 1}^{\ln 2} = \ln|u|\Big|_0^{\ln 2} = \ln(\ln 2) - \underbrace{\ln 0}_{\text{undefined!}}$$

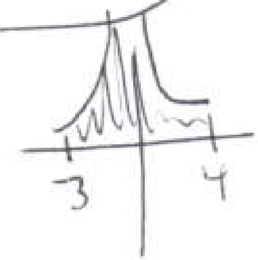
$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int_1^2 \frac{dx}{x\ln x} = \int_{\ln 1}^{\ln 2} \frac{1}{u}\,du = \underbrace{\int_0^{\ln 2} \frac{du}{u}}_{\text{improper}} = \lim_{t\to 0^+}\int_t^{\ln 2} \frac{du}{u} = \lim_{t\to 0^+} \ln|u|\Big|_t^{\ln 2}$$

$$= \lim_{t\to 0^+}\left(\ln(\ln 2) - \ln t\right) = \infty \Rightarrow \int_1^2 \frac{dx}{x\ln x} \text{ diverges.}$$

$\{\ln(t\text{ small}) = -\text{big}\}$



$\int_{-3}^{4} \frac{dx}{x^2}$ also diverges

$$\int_{-3}^{4} \frac{dx}{x^2} = \int_{-3}^{4} x^{-2}\,dx \neq \frac{x^{-3}}{3}\Big|_3^4 = \frac{4^{-3}}{-3} - \frac{(-3)^{-3}}{-3}$$

$\frac{1}{x^2}$ not continuous at 0

$\int_{-3}^{4} \frac{dx}{\sqrt[3]{x}}$ converges, though.

$$\int_0^1 \frac{dx}{x^p}\begin{cases}\text{converges: } p<1\\ \text{diverges: } p\geq 1\end{cases} \qquad \int_1^\infty \frac{dx}{x^p}\begin{cases}\text{converges: } p>1\\ \text{diverges: } p\leq 1\end{cases}$$

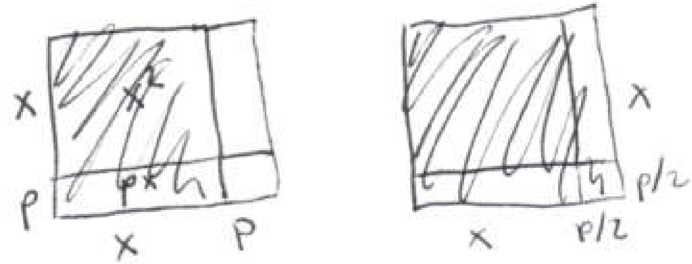
$$\sum_{n=1}^\infty \frac{1}{n^p}\begin{cases}\text{converges: } p>1\\ \text{diverges: } p\leq 1\end{cases} \qquad \int_0^\infty e^{ax}\,dx\begin{cases}\text{converges: } a<0\\ \text{diverges: } a\geq 0\end{cases}$$

$$\sum_{n=0}^\infty r^n\begin{cases}\text{converges: } |r|<1\\ \text{diverges: } |r|\geq 1\end{cases} \longrightarrow \text{(Think } e^a = |r|.)$$

Appears in integral	use: $a>0$	
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \Rightarrow a\cos\theta = \sqrt{a^2-x^2}$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow a\sec\theta = \sqrt{a^2+x^2}$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$\left.\begin{array}{l} 0 \le \theta < \frac{\pi}{2} \\ \text{or } \pi \le \theta < \frac{3\pi}{2} \end{array}\right\} \Rightarrow a\tan\theta = \sqrt{x^2-a^2}$

$$\int \frac{dx}{\sqrt{5-3x-x^2}}$$

Complete the square $x^2+px = \left(x+\frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2$



$$x^2+3x = \left(x+\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2$$

$$5-3x-x^2 = 5-(x^2+3x) = \underset{\downarrow \atop \frac{20}{4}}{5} - \left[\left(x+\frac{3}{2}\right)^2 - \frac{9}{4}\right] = \frac{11}{4} - \left(x+\frac{3}{2}\right)^2$$

$$= \int \frac{dx}{\sqrt{\left(\sqrt{\frac{11}{4}}\right)^2 - \left(x+\frac{3}{2}\right)^2}} = \int \frac{du}{\sqrt{a^2-u^2}} = \int \frac{a\cos\theta\, d\theta}{a\cos\theta} = \int d\theta = \theta + C = \underbrace{\sin^{-1}\frac{u}{a}}_{\text{solve } u = a\sin\theta \text{ for } \theta} + C$$

$du = 1\,dx$

$u = x + \frac{3}{2}$

$a = \sqrt{\frac{11}{4}} = \frac{\sqrt{11}}{2}$

$u = a\sin\theta$

$du = a\cos\theta\, d\theta$

$$\boxed{\sin^{-1}\left(\frac{x+3/2}{\frac{\sqrt{11}}{2}}\right) + C}$$

Partial fractions

$$\int \frac{(x^6-2)\,dx}{(x^2+3)^2(x-4)(x-5)^2}$$

$$\frac{\overbrace{x^6-2}^{\text{degree } 6}}{\underbrace{(x^2+3)^2(x-4)(x-5)^2}_{\text{degree } 7}} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{(x^2+3)^2} + \frac{E}{x-4} + \frac{F}{x-5} + \frac{G}{(x-5)^2}$$

$$x^6-2 = (Ax+B)(x^2+3)(x-4)(x-5)^2 + (Cx+D)(x-4)(x-5)^2 + E(x^2+3)^2(x-5) + F(x^2+3)^2(x-4)(x-5) + G(x^2+3)^2(x-4)$$

Plug in 7 different values of x and solve the resulting equations.

$$\int_3^4 \frac{dx}{x^2-x} = \int_3^4 \frac{dx}{x(x-1)} = \int_3^4 \left(\frac{-1}{x} + \frac{1}{x-1}\right)dx = \left(-\ln|x| + \ln|x-1|\right)\Big|_3^4$$

$$= \ln\left|\frac{x-1}{x}\right|\Big|_3^4 = \ln\frac{3}{4} - \ln\frac{2}{3} = \ln\frac{3/4}{2/3} = \boxed{\ln\frac{9}{8}}$$