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Today: part 2 review
- Some of ch. 7
- ch 6
- ch 8

Wed: Chapter 9 + 10

Thurs: Chapter 11

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* Long division

$$\int \frac{x^2\,dx}{x-3} \quad \begin{matrix} \} \text{degree } 2 \\ \} \text{degree } 1 \end{matrix}$$

top degree $\geq$ bottom degree $\Rightarrow$ divide

$$\begin{array}{r} x+3 \\ x-3 \,\big)\, x^2 \\ -(x^2-3x) \\ \hline +3x \\ -(3x-9) \\ \hline +9/_{x-3} \end{array}$$

$$\Rightarrow \int \frac{x^2\,dx}{x-3} = \int \left(x+3+\frac{9}{x-3}\right) \Rightarrow \frac{x^2}{2} + 3x + 9\int \frac{dx}{x-3} \Rightarrow \frac{x^2}{2} + 3x + 9\int \frac{du}{u} \Rightarrow \boxed{\frac{x^2}{2} + 3x + 9\ln|x-3| + C}$$

$u = x-3$

$du = dx$

* trig integrals

- $\int \sin^4 x \cos^{7} x\,dx = \int \sin^4 x \overbrace{(1-\sin^2 x)^3}^{(\cos^2 x)^3} \underbrace{\cos x\,dx}_{du} = \int u^4 (1-u^2)^3\,du$

  $u = \sin x$

- $\int \cos^6 x \sin^{5} x\,dx = \int \cos^6 x (1-\cos^2 x)^2 \underbrace{\sin x\,dx}_{-du} \Rightarrow \int u^6 (1-u^2)^2 (-du)$

  $u = \cos x$

- $\int \sin^{2} x \cos^{4} x\,dx$

  use: $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ + $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\int \tan^6 x \sec^{8} x\,dx$ → good $= \int \tan^6 x \underbrace{(1+\tan^2 x)^3}_{(\sec^2 x)^3} \underbrace{\sec^2 x\,dx}_{du} = \int u^6 (1+u^2)^3\,du$

$u = \tan x$

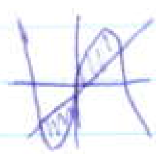
- $\int \sec^5 x \tan^{3} x\,dx = \int \sec^4 x \underbrace{(\sec^2 x - 1)}_{(\tan^2 x)} \underbrace{\sec x \tan x\,dx}_{du}$ → $u^4(u^2-1)\,du$ → $\int (u^6 - u^4)\,du \Rightarrow \frac{u^7}{7} - \frac{u^5}{5} + C$

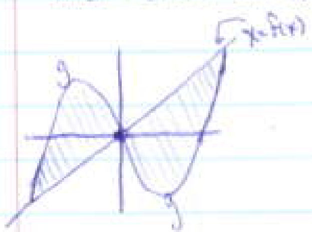
  $\boxed{\frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C}$

  $u = \sec x$

* (Ch. 6)

Area between curves $y = x,\; y = 3x - x^3$:





solve $x = 3x - x^3$:

$0 = 2x - x^3 \Rightarrow 0 = x(2-x^2) \Rightarrow 0 = x(\sqrt{2}-x)(\sqrt{2}+x)$

$f = g \quad g \neq f \quad f = g \quad f < g \quad f = g$

$-\sqrt{2} \quad -1 \quad 0 \quad 1 \quad \sqrt{2}$ $\qquad x = 0, \pm\sqrt{2}$

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$f = -1 \qquad f = 1$

$g = -2 \qquad g = 2$

$$\text{Area} = \int_{-\sqrt{2}}^{0} (f - g)\,dx + \int_{0}^{\sqrt{2}} (g - f)\,dx$$

* **Volume** : slices : $V = \int dV_{slice} = \int A_{slice}\,dx$ (x-formula) or $\int A_{slice}\,dy$ (y-formula)

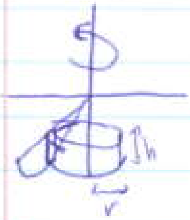
revolve between the region

$y = x$, $y = 3x - x^3$, $x = 0$, $x = -\sqrt{2}$ about x-axis

$$A_{slice} = \pi(R^2 - r^2) = \pi\left((3x - x^3)^2 - x^2\right)$$

$$V = \int_{-\sqrt{2}}^{0} \pi\left((3x - x^3)^2 - x^2\right)$$

* **Shells** : Revolve same region about y-axis :



$$V = \int dV_{shell} = \int 2\pi r h\,dr$$

$$V = \int dV_{shell} = \int 2\pi r h\,dr \qquad V = \int_{0}^{\sqrt{2}} 2\pi(-x)\left(x - (3x - x^3)\right)(-dx)$$

$r = -x \qquad h = x - (3x - x^3) \qquad dr = -dx$

* Chapter 8 : Arc length + surface area

$ds^2 = dx^2 + dy^2$

$ds = \sqrt{dx^2 + dy^2}$

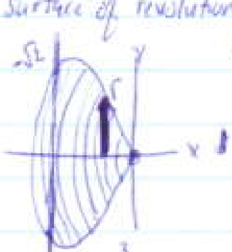
for $y = f(x)$: $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

for $x = g(y)$: $ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$

Example: length of curve $x = 3y - y^3$ from $y = -2$ to $y = -1$ is:

$$\int_{-2}^{-1} \sqrt{1 + ((3y - y^3)')^2}\,dy = \int_{-2}^{-1} \sqrt{1 + (3 - 3y^2)^2}\,dy$$

for surface of revolution



$$A = \int 2\pi r\,ds$$

$$ds = \sqrt{1 + (3 - 3x^2)^2}\,dx$$

$$A = \int_{-\sqrt{2}}^{0} 2\pi(x^3 - 3x)\,dx$$

$y = 3x - x^3 = -r$

One more thing from Ch. 7:

$$\int \sin 4x \cos 5x \, dx = ?$$

$$= \int \frac{1}{2}\left[\sin(4x-5x) + \sin(4x+5x)\right] dx$$

$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$

$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$

$$= \frac{1}{2}\int (\sin(-x) + \sin 9x)\,dx = \frac{1}{2}\int (\sin 9x - \sin x)\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos 9x}{9} + \cos x\right) + C$$