Today Review Ch 9, 10

9.3 Seperable Differential Equation
9.5 Linear

Seperable put equation into form.

$f(x)\,dx = g(y)\,dy \qquad f(x)\,dx + g(y)\,dy = 0$

$$\int f(x)\,dx + \int g(y)\,dy = \int 0 = C$$

Ex.

① $\dfrac{dy}{dx} = \dfrac{5}{xy^2}$

② $y^2\,dx\,\dfrac{dy}{dx} = \dfrac{5}{xy^2} \cdot y^2\,dx$

③ $y^2\,dy = \dfrac{5\,dx}{x}$

④ $\int y^2\,dy = 5\int \dfrac{dx}{x}$

⑤ $\boxed{\dfrac{y^3}{3} = 5\ln|x| + C}$

If were also given $y=7$ when $x=e$
then we can solve for C

(6) $$\frac{7^3}{3} = 5\ln e + C \Rightarrow \frac{7^3}{3} - 1 = C$$

(7) $$\frac{y^3}{3} = 5\ln|x| + \frac{7^3}{3} - 1$$

(8) $$y = \sqrt[3]{15\ln|x| + 7^3 - 3}$$

Linear: Can be put in form

$$\underbrace{y' + p'y = q}_{\frac{dy}{dx} + p'(x)y = q(x)}$$

$f(0)=4$ and $f'(x) = 3x + f(x)$

Find $f(x)$

$$f' \underbrace{-1}_{p'} f = \underbrace{3x}_{q} \qquad p' = -1, \quad p = -x$$

$\cancel{u = pf}$ $\qquad u = e^p f$

$$u' = p'e^p f + e^p f' = e^p \underbrace{(f' + p'f)}_{q}$$

Summary: $u = e^p f$

$u' = e^p q$

$u = e^{p} f = e^{-x} f$

$u' = e^{p} q = e^{-x}(3x)$

$g = G$

Ex II

1 $u = 3\int \underbrace{x}_{F} \underbrace{e^{-x}}_{g'} dx$ 2 $= 3(Fg - \int g F' dx$

$F' = 1$

$G = -e^{-x}$

3 $3(x(-e^{-x}) - \int (e^{-x})(1) dx)$

4 $3(-xe^{-x} - e^{-x}) + c$

5

$e^{-x} f = u = -3(x+1)e^{-x} + C$

6 $f = e^{x} u = -3(x+1)e^{-x}e^{x} + Ce^{x}$

7 $= \boxed{-3(x+1) + Ce^{x}}$

$y = f(x)$

Kelly's question

$\frac{dy}{dx} = f'(x) = 3x + y$

$\frac{dy}{dx} + (-1)y = 3x$

$u = e^{\int (-1) dx} y$

$u = e^{-x} y$

Cont. Ex II

$$f(0) = 4 = -3(0+1) + ce^0 = -3 + c$$

$$C = 7$$

$$= f(x)$$

$$\boxed{-3(x+1) + 7e^x}$$

Chapter 10
Parametric Equations

$x = f(t)$
$y = g(t)$

Special Case: Polar Equ.

$r = f(\theta)$

$\Rightarrow = \begin{cases} x = f(\theta)\cos\theta \\ y = f(\theta)\sin\theta \end{cases}$

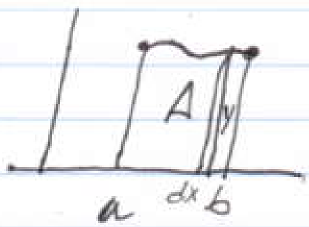
because $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$m = \frac{dy}{dx}$ = slope of a tangent line

$$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dm}{dx} = \frac{dm/dt}{dx/dt}$$

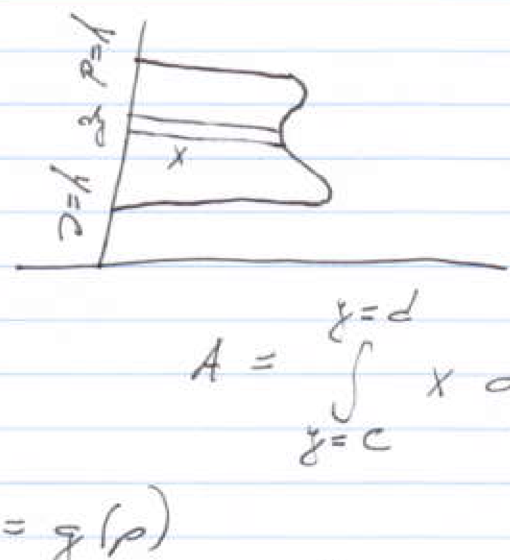
$\begin{cases} >0 \Rightarrow (CU) \\ <0 \Rightarrow (CD) \end{cases}$



$$A = \int_{x=a}^{x=b} y\,dx$$

$x = f(t)$
$y = g(t)$
$a = f(p)$
$b = f(q)$

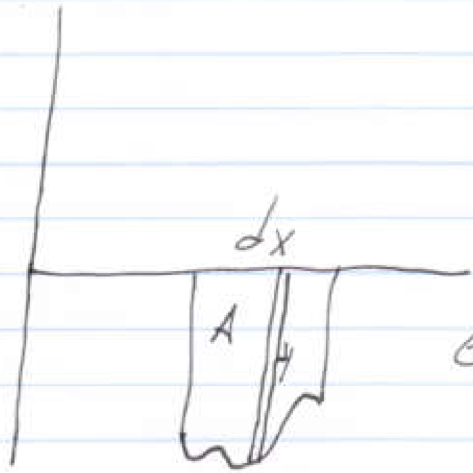
$$A = \int_{t=p}^{t=q} \underbrace{g(t)}_{y} \underbrace{f'(t)\,dt}_{dx}$$



$$A = \int_{y=c}^{y=d} x\,dy$$

$c = g(p)$
$d = g(q)$
$x = f(t)$
$y = f(t)$

$$\int f(t)\, g'(t)\, dt$$



$\leftarrow$ y is negative

$$A = \int (-y)\,dx$$

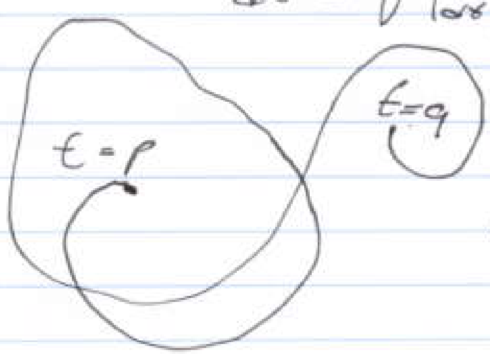
Arc Length

$$ds^2 = dx^2 + dy^2$$

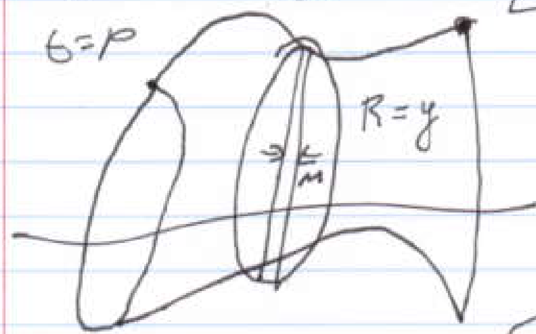
$$\frac{ds^2}{dt^2} = \frac{dx^2}{dt^2} + \frac{dy^2}{dt^2}$$

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$



$$\text{length} = \int_{t=p}^{t=q} \sqrt{(f'(t))^2 + (g'(t))^2}\,dt$$



$$V_{slice} = \pi R^2 dx$$
$$= \pi y^2 dx$$

Volume?

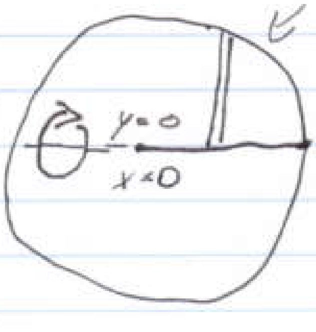
$$\boxed{V = \int_{x=a}^{x=b} \pi y^2 dy}$$

equal

$$\boxed{\int_{t=p}^{t=q} \pi (g(t))^2 f'(t) dt}$$

Prove that a sphere of radius 1 has surface area 4π

you get a ring out of slice



$$A_{ring} = 2\pi R ds$$

$$A = \int_{\theta=0}^{\theta=\pi} 2\pi \sin\theta \, ds$$

$$ds = \sqrt{(\cos'\theta)^2 + (\sin'\theta)^2}\ d\theta$$

$$= \sqrt{(-\sin\theta)^2 + (\cos\theta)^2}\ d\theta$$

$$= \sqrt{\sin^2\theta + \cos^2\theta}\ d\theta$$

$$= \sqrt{1} = 1$$

$$A = \int_{\theta=0}^{\theta=\pi} 2\pi \sin\theta\, d\theta$$

$$2\pi(-\cos\theta)\Big|_{\theta=0}^{\theta=\pi}$$

$$2\pi(-(-1)-(-1)) = 4\pi$$

Elimination of the Parameter

$$\begin{bmatrix} x = \cos\theta \\ y = \sin\theta \end{bmatrix} = x^2 + y^2 = 1$$

$$x = 5e^{-t}$$
$$y = 2 - e^{3t}$$

$$e^{-t} = \frac{x}{5} \rightarrow e^{t} = \frac{5}{x} \rightarrow e^{3t} = \frac{5}{x^3}$$

$$y = 2 - \left(\frac{5}{x}\right)^3$$

Polar plotting:

(r, θ) is found by starting

at $x=r$ + $y=0$ and rotating θ radians

counterclockwise