4-6-10

To Day: Review CH. 11

Final: Here, Monday 11 A.M - 2 P.M
Bring $\leq$ 3 sheets of notes
No Book No calculator

Sequences: $a_0, a_1, a_2, a_3 \ldots = \{a_n\}_{n\geq 0}^{\infty}$

Series: $a_0+a_1+a_2+a_3+\ldots \sum_{n=0}^{\infty} a_n = \lim_{m\to\infty} \sum_{n=0}^{m} a_n$

$= \lim_{m=\infty} (a_0+a_1+a_2+a_3+\ldots+a_m)$

Divergent Test

$\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum_{n=0}^{\infty} a_n$ Diverges

Geometric Series

$$\sum_{n=0}^{\infty} r^n = \begin{cases} \frac{1}{1-r} : & |r|<1 \\ \text{Diverges} & |r| \geq 1 \end{cases}$$

Integral Test

If $f(x)$ is eventually positive and Decreasin

then $\sum_{n=0}^{\infty} f(n$ & $\int_0^{\infty} f(x)\,dx$

Either Both Divege or Both converge

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{ COMPARISON TEST
{ LIMIT COMPARISON TEST (L.C.T)

ALTERNATING SERIES TEST
RATIO TEST
Root TEST

$\sum_{n=5}^{\infty} \frac{n}{n^5-7n^4}$ COMPARE TO $\frac{n}{n^5} \left(= \frac{1}{n^4}\right)$

LCT SAY IF a_n, b_n ARE EVENTUALLY ALWAYS >0 AND $0 < \lim_{n\to\infty} \frac{a_n}{b_n} < \infty$, then $\sum_{n=0}^{\infty} a_n$ & $\sum_{n=0}^{\infty} b_n$

both CONVERGE OR both Diverge.

$\lim_{n\to\infty} \frac{n}{n^5-7n^4} = \lim \frac{1/n^4}{1-7/n} = 0$ TELLS US NOTHING ABOUT CONVERGE OF $\sum_{n=5}^{\infty} \frac{n}{n^5-7n^4}$

$\lim_{n\to\infty} \frac{1}{n} = 0 = \lim_{n\to\infty} \frac{1}{n^2}$, but

$\sum_{n=1}^{\infty} \frac{1}{n^1} = \sum_{n=1}^{\infty} \frac{1}{n} = \infty > \frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

$\sum_{n=3}^{\infty} \frac{1}{n \ln n}$ use INTEGAL TEST:

FOR ALL $n \geq$: $\frac{1}{n} \leq \frac{\ln n}{n}$, so $\infty = \sum_{n=3}^{\infty} \frac{1}{n} \leq \sum_{n=3}^{\infty} \frac{\ln n}{n}$

$$\Rightarrow \sum_{n=3}^{\infty} \frac{\ln n}{n} = \infty$$

$$\int_3^{\infty} \frac{dx}{x \ln x} = \int_{\ln 3}^{\infty} \frac{du}{u} = \lim_{t\to\infty} \int_{\ln 3}^{t} \frac{du}{u} = \lim_{t\to\infty} \ln|u| \Big|_{\ln 3}^{t}$$

$$u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{dx}{x}$$

$$= \lim_{t\to\infty} \underbrace{\ln(t)}_{\infty} - \underbrace{\ln(\ln 3)}_{\text{CONSTANT}} = \infty$$

So, $\int_3^{\infty} \frac{dx}{x \ln x}$ diverges. therefor $\sum_{n=3}^{\infty} \frac{1}{n \ln n}$ Diverges Too

on the other hand $\sum_{n=3}^{\infty} \frac{1}{n (\ln n)^2}$ CONVERGES

(Use INTEGAL TEST)

Quicker to note : $\int_a^{\infty} \frac{dx}{x^p}$ converges IF $P > 1$

Diverges IF $P \leq 1$

a = ANY positive CONSTANT

Power SERIES

$$\sum_{n=1}^{\infty} \frac{(x-7)^{3n}}{8^n n}$$

For what x Does this CONVERGE ?

$$\frac{|x-7|^3}{8} < 1 \iff |x-7| < \sqrt[3]{8} = 2$$

$$\iff -2 < x-7 < 2$$

2 IS CALLED the RADIUS OF CONVEGENCE

$\cancel{x = c \Rightarrow}\; a_n$ = TAYLOR SERIES

MACLAURIN SERIES

$$(T)\; f(x) = \sum_{n=0}^{\infty} (f^{n}(a)\,\frac{(x-a)^n}{n!}$$

$$(M.)\; f(x) = \sum_{n=0}^{\infty} f^{n}(0)\,\frac{x^n}{n!}$$

$$\frac{\frac{a}{b}}{\frac{c}{D}} = \frac{a/c}{b/D}$$

Ratio

$$\lim \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n=\infty} \left| \frac{\frac{(x-7)^{3(n+1)}}{8^{n+1}(n+1)}}{\frac{(x-7)^{3n}}{8^n n}} \right|$$

$$= \lim_{n\to\infty} \left|\frac{(x-7)^3}{8\left(\frac{n+1}{n}\right)}\right| = \lim_{n\to\infty} \frac{|x-7|^3}{8\left(1+\frac{1}{n}\right)} = \underbrace{\frac{|x-7|^3}{8}}_{\text{Call this } L.}$$

The Ratio Test Says

$L > 1 \Rightarrow \sum_{n=1}^{\infty}$ Diverges

$L < \Rightarrow \underbrace{\sum_{n=1}^{\infty} |a_n| \text{ converges}}_{\text{Absolute Convergence}} \Rightarrow \sum_{n=1}^{\infty} a_n$ conveges

For what x Does this converge?

It converges on $(5, 9)$.

It Diverges on $(-\infty, 5) \cup (9, \infty)$

What About 5 & 9 ?

conveges   Diveges

Plug in 5 & 9
Use Other Tests

$x = 5 \Rightarrow a_n = \frac{(-1)^n}{n} \cdot \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CONVERGES BY AST

$x = 9 \Rightarrow a_n = \frac{1}{n} \cdot \sum_{n=1}^{\infty} \frac{1}{n}$ Diverges

p-SERIES

$p = 1$

TAYLOR SERIES of $\sin x$ at $a = \pi/2$

$f(x) = \sin x$

n	$f^{(n)}(x)$	$f^{(n)}(a)$
0	$\sin x$	$\sin \pi/2 = 1$
1	$\sin' x = \cos x$	$\cos(\pi/2) = 0$
2	$\cos' x = -\sin x$	$-\sin(\pi/2) = -1$
3	$-\sin' x = -\cos x$	$-\cos(\pi/2) = 0$
4	$-\cos' x = \sin x$	$\sin(\pi/2) = 1$

$$\sin x = 1 + \frac{0}{1!}\left(x - \frac{\pi}{2}\right) + \frac{-1}{2!}\left(x - \frac{\pi}{2}\right)^2$$

$$+ \frac{0}{3!}\left(x - \frac{\pi}{2}\right)^3 + \frac{1}{4!}\left(x - \frac{\pi}{2}\right)^4 + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{\left(x - \frac{\pi}{2}\right)^{2n}}{(2n)!}$$

$$\sin^{(n)}\left(\frac{\pi}{2}\right) = \begin{cases} 0: & n \text{ IS ODD} \\ 1: & n = 0, 4, 8, 12, 16 \ldots \\ -1: & n = 2, 6, 10, 14, 18 \ldots \end{cases}$$