$\sum$ notation

(Similar to Keisler 4.6)

$1+2+3+4+5+\cdots+15$

sum of k as k goes from 1 to 15 (over the integers)

$$\sum_{k=1}^{15} k = 1+2+3+4+5+\cdots+15$$

$$\sum_{k=-2}^{3} k^2 = (-2)^2+(-1)^2+0^2+1^2+2^2+3^2$$

$$\sum_{j=-2}^{3} j^2 = (-2)^2+(-1)^2+0^2+1^2+2^2+3^2$$

$$\sum_{j=-2}^{3} k^2 = k^2+k^2+k^2+k^2+k^2+k^2$$

$$\sum_{j=-2}^{3} 5 = 5+5+5+5+5+5$$

HW: # 1 (due tomorrow in class)

Express the following in Σ-notation:

the sum of the reciprocals of the first ten perfect cubes (starting at 1).

$$\sum_{k=0}^{4} (-1)^k = (-1)^0 + (-1)^1 + (-1)^2 + (-1)^3 + (-1)^4$$

$$= \cancel{1} + \cancel{(-1)} + \cancel{1} + \cancel{(-1)} + 1$$

$$= \boxed{1}$$

$$2 - 3 + 4 - 5 + 6 - 7 + 8 - 9$$

$$= \left[(-1)^2 2\right] + \left[(-1)^3 3\right] + \left[(-1)^4 4\right] + \cdots \left[(-1)^9 9\right] = \sum_{k=2}^{9} (-1)^k k$$

$$\sum_{k=2}^{2^{99}+1} (-1)^k k = ?$$

$$\sum_{k=2}^{2^{99}+1} (-1)^k k = \overbrace{2-3}^{-1} + \overbrace{4-5}^{-1} + \overbrace{6-7}^{-1} +$$

$$\cdots \left.\begin{matrix} +(2^{99}-2) \\ \cancel{8} -(2^{99}-1) \end{matrix}\right\} -1 \quad \begin{matrix} \leftarrow \text{even} \\ \leftarrow \text{odd} \end{matrix}$$

$$\left.\begin{matrix} +(2^{99}) \\ -(2^{99}+1) \end{matrix}\right\} -1 \quad \begin{matrix} \leftarrow \text{even} \\ \leftarrow \text{odd} \end{matrix}$$

How many -1's ?

Simpler case: 6 #s $\quad 6 = 7-2+1$

$$\overbrace{\underbrace{2-3}_{-1} + \underbrace{4-5}_{-1} + \underbrace{6-7}_{-1}}$$

$6/2 = 3$ $\quad -1$'s

$$\underbrace{a,\ a+1,\ a+2,\ \ldots,\ b-2,\ b-1,\ b}_{b-a+1 \text{ #s}}$$

$(2^{99}+1)-2+1$ #5

2^{99} #5

$2^{99}/2 = 2^{98}$ -1's

$$\sum_{k=2}^{2^{99}+1} (-1)^k k = -2^{98}$$

HW #2

Compute $$\sum_{j=0}^{1000} (-1)^{3j}(2j+1)$$

HW #3 Look up formulas for $\sum_{k=1}^{n} k$, $\sum_{k=1}^{n} k^2$, and $\sum_{k=1}^{n} k^3$

and use them to evaluate

$$\sum_{k=1}^{50} (1+3k-2k^2+4k^3).$$

Hint: look up "summation"...