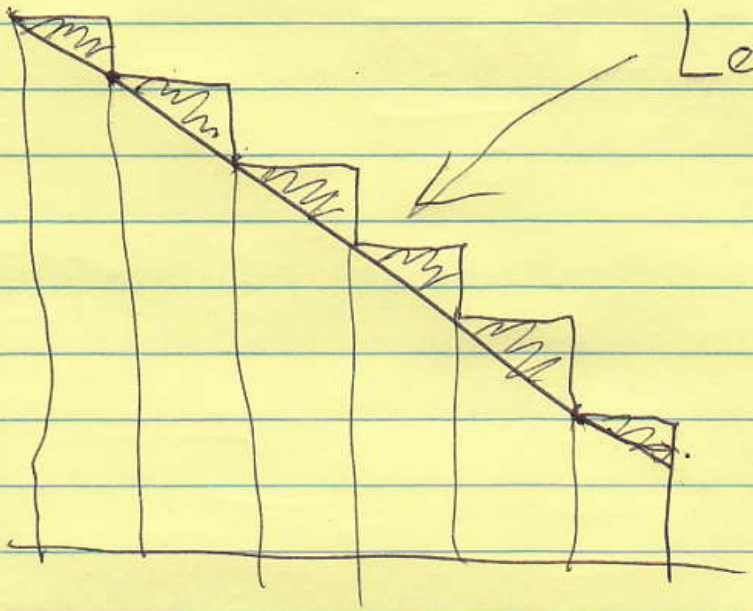
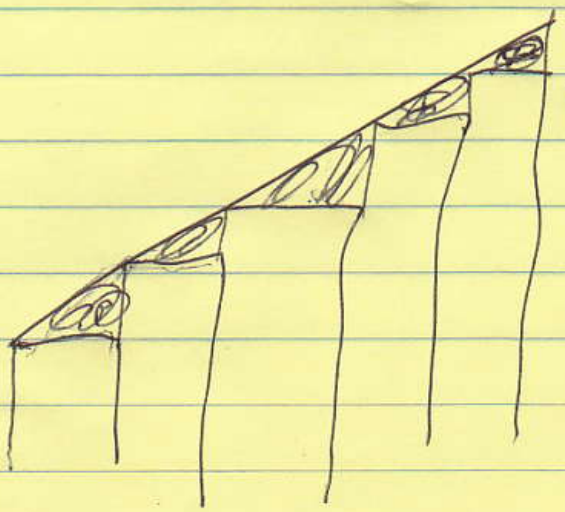
Last time



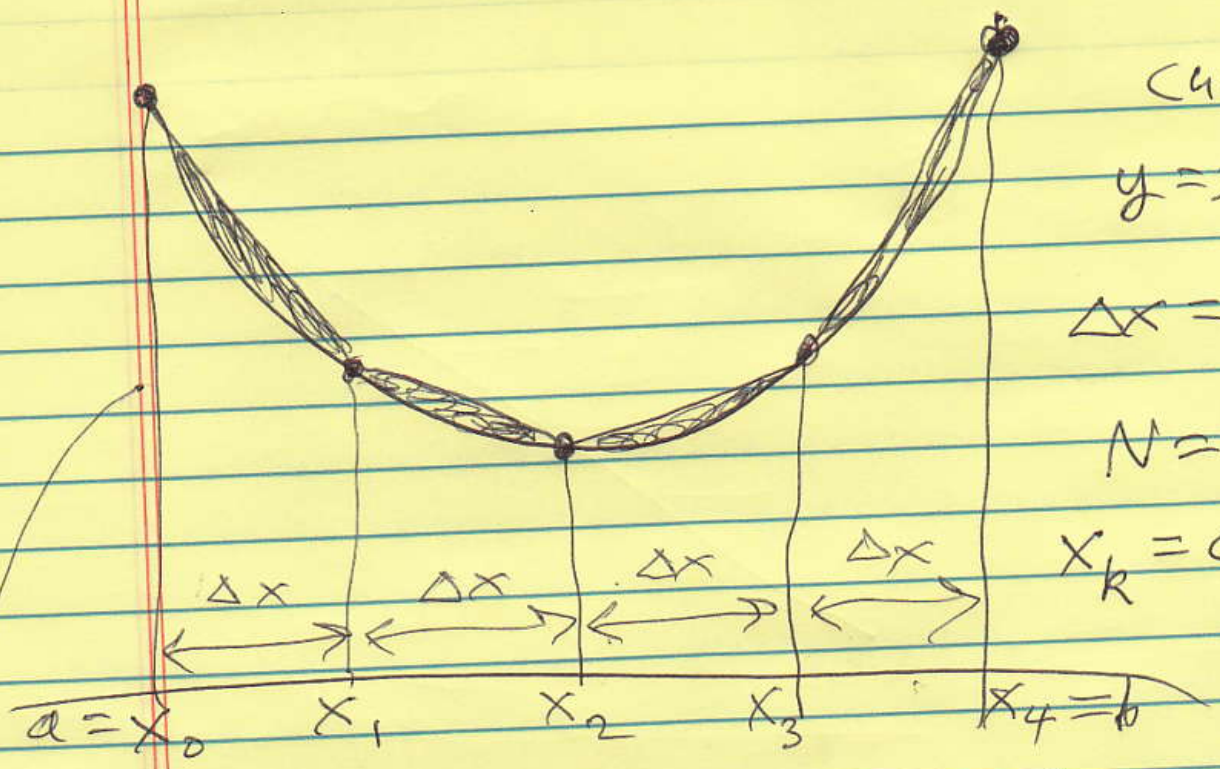
Left-endpoint rule overestimates $\int_a^b f(x)\,dx$ when $f \searrow$

right-endpoint rule underestimates when $f \searrow$

When $f \nearrow$, L.E. underestimates



And R.E. overestimates



curve

$y = f(x)$

$\Delta x = \frac{b-a}{N}$

$N = 4$

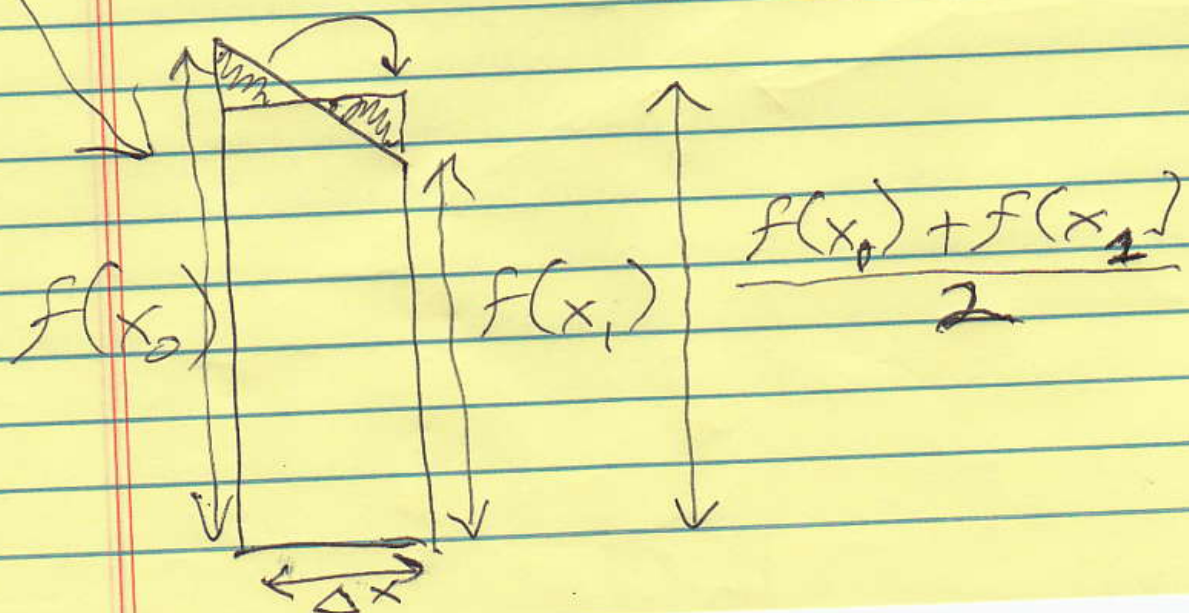
$x_k = a + k\Delta x$

When f is (C.U.) (concave up)

the trapezoid rule overestimates

When f is (CD) (concave down)

the trapezoid rule underestimate



$\frac{f(x_0) + f(x_1)}{2}$

Area of kth trapezoid:

$$\frac{f(x_{k-1}) + f(x_k)}{2} \Delta x$$

Total: $\left(\sum_{k=1}^{4} \frac{f(x_{k-1}) + f(x_k)}{2} \right) \Delta x$

$$= \left[\left(\frac{f(x_0)}{2} + \frac{f(x_1)}{2} \right) + \left(\frac{f(x_1)}{2} + \frac{f(x_2)}{2} \right) + \left(\frac{f(x_2) + f(x_3)}{2} \right) + \left(\frac{f(x_3) + f(x_4)}{2} \right) \right] \cdot \Delta x$$

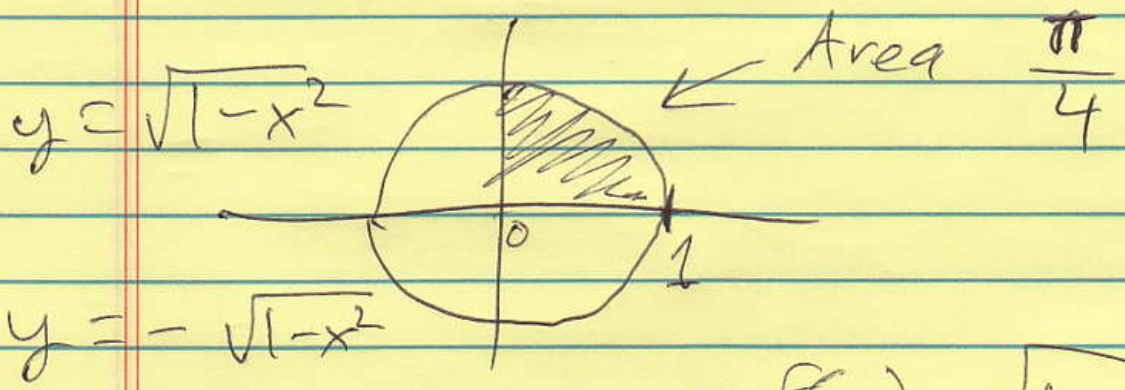
$$= \frac{\Delta x}{2} \left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4) \right]$$

circle : radius 1,

center (0, 0)

$$x^2 + y^2 = 1^2$$

$$y = \pm\sqrt{1-x^2}$$



$$f(x) = \sqrt{1-x^2}$$

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$$

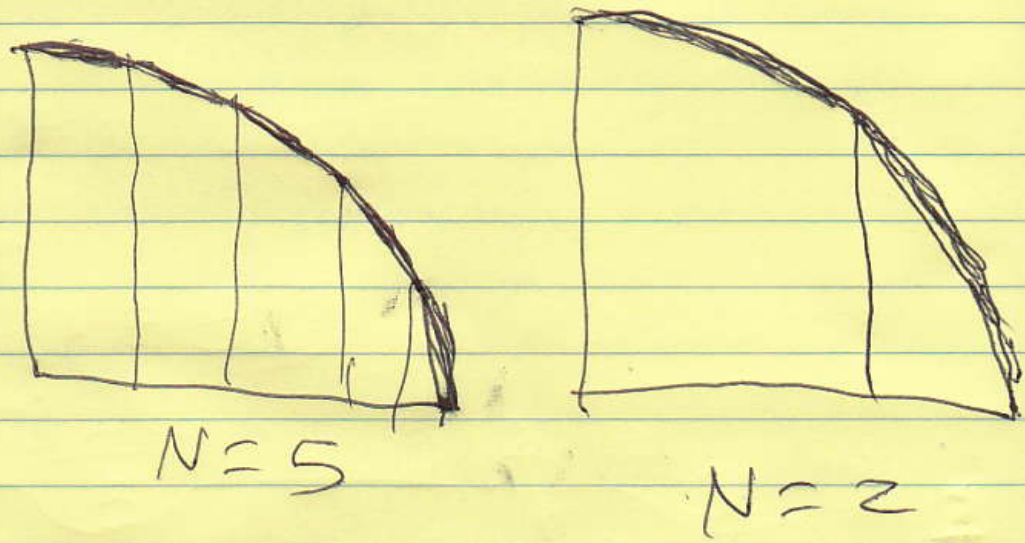
Try $N = 5$ (trapezoids)

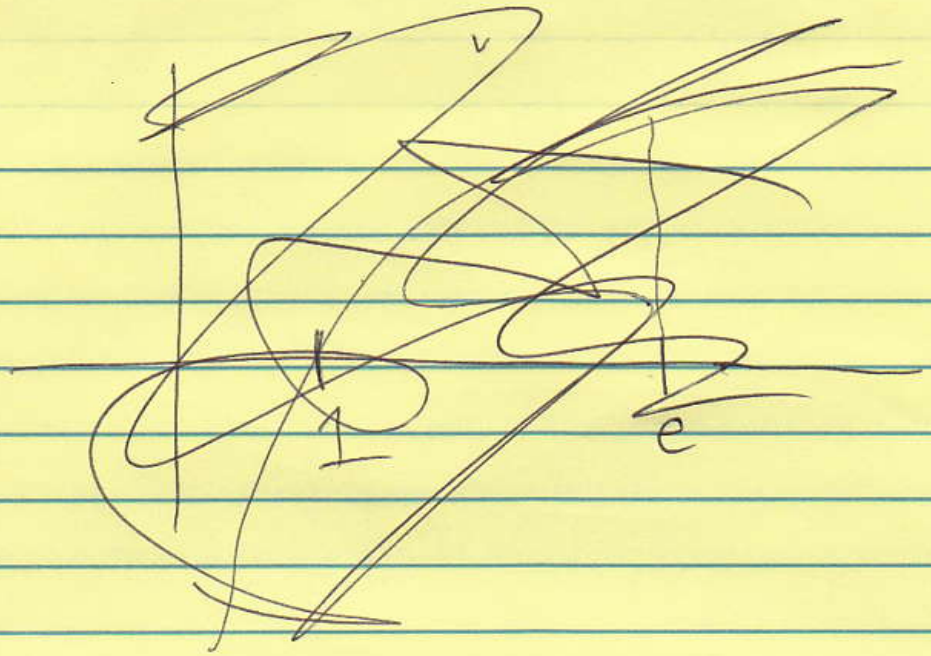
$$\Delta x = \frac{1-0}{5} = 0.2 = \frac{1}{5}$$

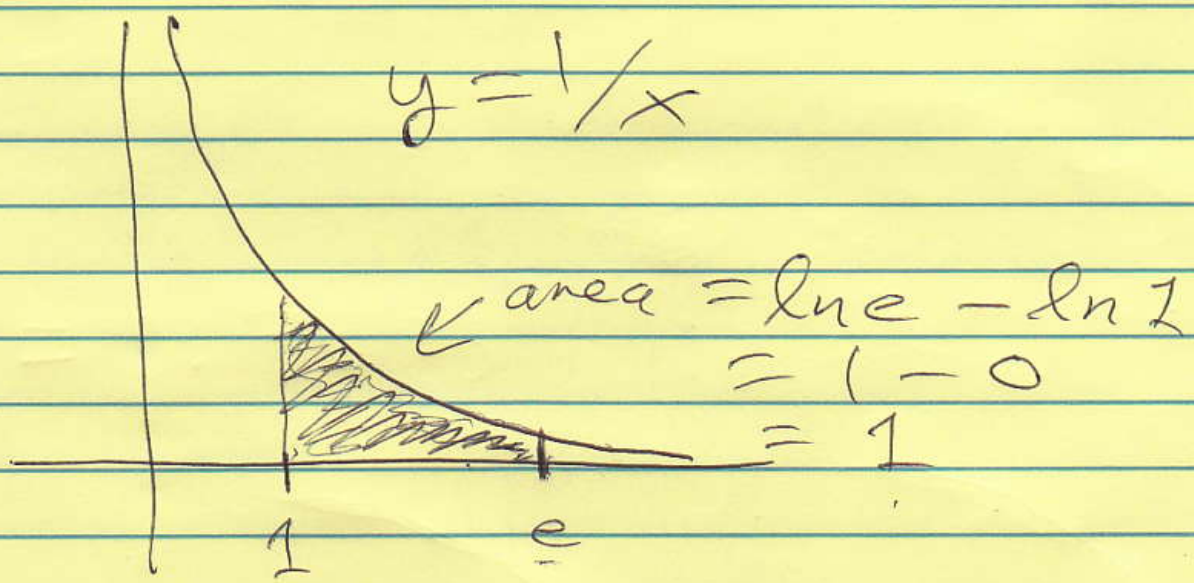
x_0	0	$f(x_0)$	$1 = 1$
x_1	0.2	$f(x_1)$	$\sqrt{.96} = .9798$
x_2	0.4	$f(x_2)$	$\sqrt{.84} = .9165$
x_3	0.6	$f(x_3)$	$\sqrt{.64} = .8$
x_4	0.8	$f(x_4)$	$\sqrt{.36} = .6$
x_5	1	$f(x_5)$	$0 = 0$

$$\frac{\Delta x}{2}\Big(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_5)\Big)$$

$$= 0.7593 \quad \text{(underestimate)}$$







We'll eventually use Taylor series to estimate e.

HW

$$\int_1^2 x^3 \, dx$$

Compute this exactly and figure out how many trapezoids you need to get within 0.5 of the exact value.

$$\int_3^5 \cos(x^2)\,dx$$

$N = 20$ trapezoids

$a = 3 \quad b = 5$

$$\Delta x = \frac{b-a}{20} = \frac{2}{20} = 0.1$$

$$x_k = 3 + 0.1 \cdot k$$

$x_0 = 3 \quad x_1 = 3.1 \quad x_2 = 3.2 \cdots x_{20} = 5$

$$\frac{\Delta x}{2} \sum_{k=1}^{N} \left(\cos(x_{k-1}^2) + \cos(x_k^2) \right)$$

$$\frac{0.1}{2} \sum_{k=1}^{20} \left(\cos\left((3 + 0.1 \cdot k)^2\right) + \cos\left((3 + 0.1 \cdot (k-1))^2\right) \right)$$

$$= -0.088188$$

HW (part 2)

Write a similar formula in Σ-notation for the trapezoid estimate of

$$\int_{-1}^{0} \sqrt{1+x^3}\, dx$$ with

$N = 30$ (trapezoids).