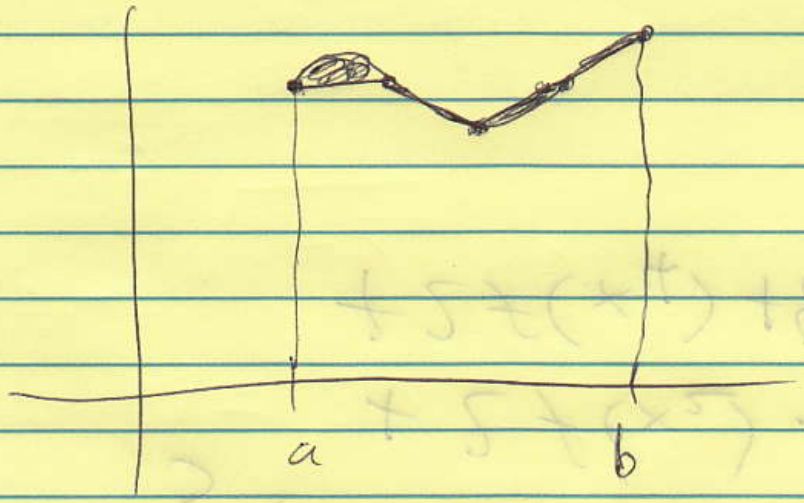
Last time: Trapezoid rule

Idea: approximate curve by lines




Simpson's Rule: approximate curves by parabolas. (Technically, "parabolic arcs.")

Estimate $\ln(10)$ using Simpson's Rule.

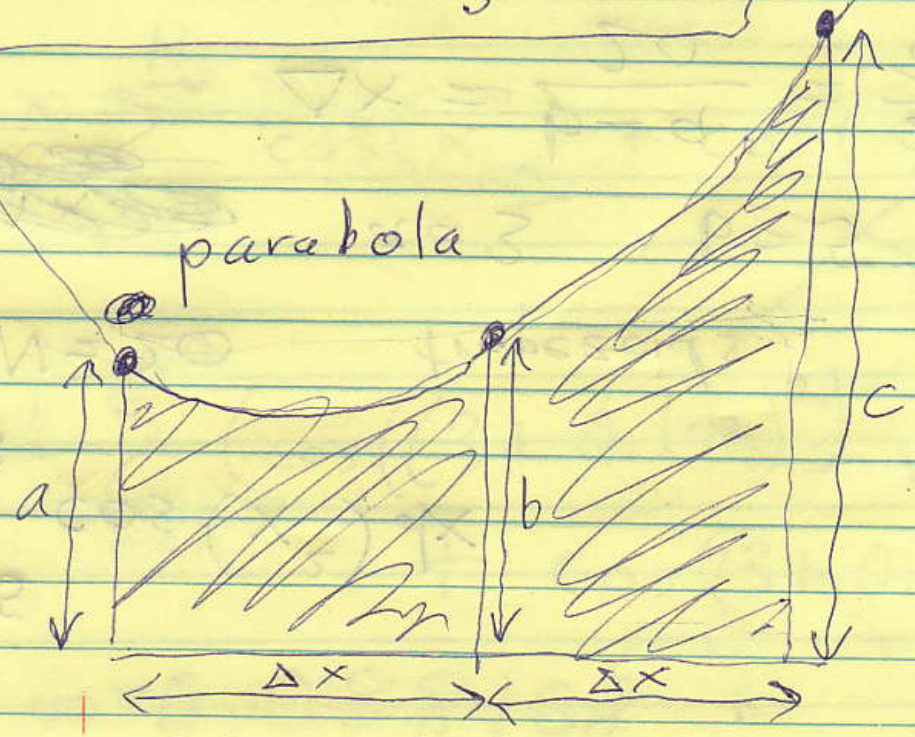
$$\int_a^b \frac{1}{x}\,dx = \ln|x|\Big|_a^b = \ln|b| - \ln|a|$$

$$= \ln\left|\frac{b}{a}\right|$$

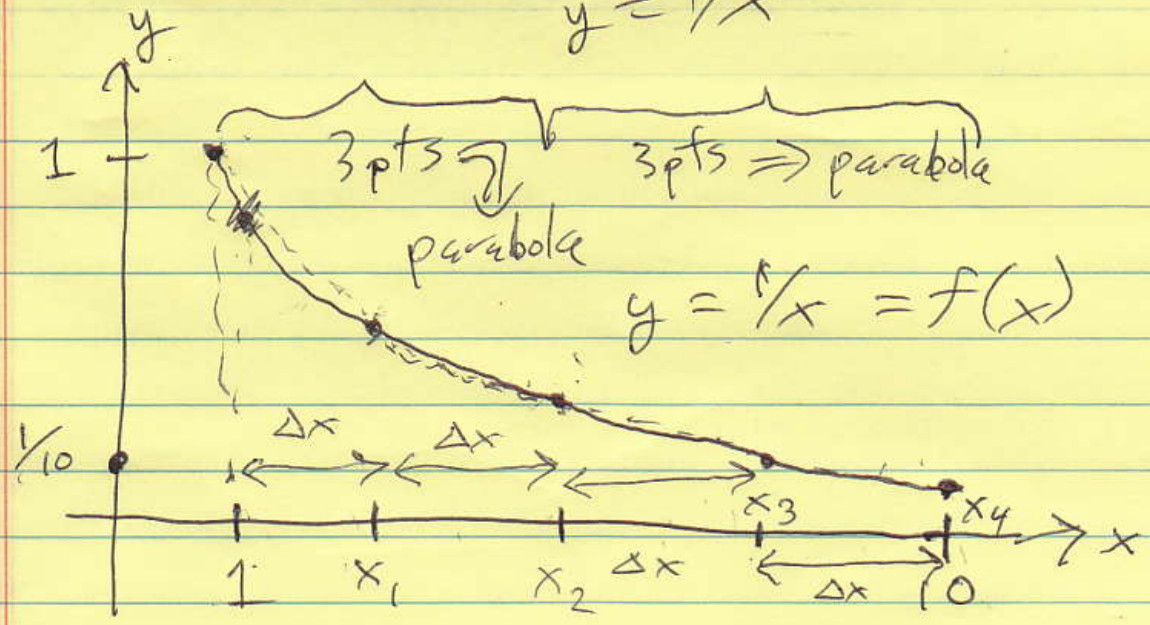
$$\int_1^{10} \frac{1}{x}\,dx = \ln\left|\frac{10}{1}\right| = \ln(10).$$

area $A = \frac{a+4b+c}{3}\Delta x$

Proof:
Keisler
4.6



$y = 1/x$




$a = 1$ $\qquad b = 10$

$x_0 = 1$

$N = 4$ $\qquad \Delta x = \frac{10-1}{4} = 2.25$

$x_k = a + k\Delta x = 1 + k \cdot 2.25$

$x_0 = 1 \rightarrow f(x_0) = 1/1 = 1.0000$

$x_1 = 3.25 \rightarrow f(x_1) = 1/3.25 = .3077$

$x_2 = 5.5 \rightarrow f(x_2) = 1/5.5 = .1818$

$x_3 = 7.75 \rightarrow f(x_3) = 1/7.75 = .1290$

$x_4 = 10 \rightarrow f(x_4) = 1/10 = .1000$

Area under 1st parabola:

$$\left(f(x_0) + 4f(x_1) + f(x_2)\right)\frac{\Delta x}{3}$$

Area under 2nd parabola:

$$\left(f(x_2) + 4f(x_3) + f(x_4)\right)\frac{\Delta x}{3}$$

$$\frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right)$$

$$= 2.4080$$

$$\ln(10) = 2.302585\ldots$$

General pattern: $N = 10$ (5 parabolas)

1 4 1

1 4 1

1 4 1

1 4 1

1 4 1

1 4 2 4 2 4 2 4 2 4 1

HW #1 Estimate $\ln 2$ using $N = 6$ (3 parabolas)
Then estimate it again with $N = 6$ trapezoids.

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Note: $N$ must be even for Simpson's Rule

$\Sigma$-notation: ~~programmable~~ programmable

Formulas for use of Simpson's Rule

Estimate $\ln(10)$ with $N=2000$ & Simpson's Rule (so 1000 parabolas)

$$\frac{\Delta x}{3}\sum_{k=1}^{1000}\left(f(x_{2k-2})+4f(x_{2k-1})+f(x_{2k})\right)$$

$$f(x)=\frac{1}{x} \qquad \Delta x=\frac{10-1}{2000}=.0045$$

$$\left(\frac{.0045}{3}\right)\sum_{k=1}^{1000}\left(\frac{1}{1+.0045(2k-2)}+\frac{4}{1+.0045(2k-1)}+\frac{1}{1+.0045(2k)}\right)$$

$$x_k=a+k\Delta x=1+.0045k$$

$$\Delta x=\frac{b-a}{N} \qquad a=1 \quad b=10 \quad N=2000$$

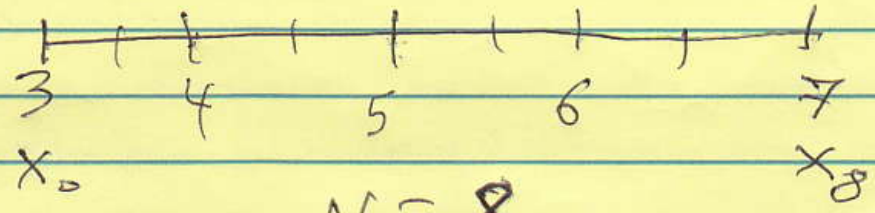
# HW # 2

| x | f(x) |
|---|---|
| 3.0 | 12.79 |
| 3.5 | 11.62 |
| 4.0 | 12.51 |
| 4.5 | 13.70 |
| 5.0 | 15.63 |
| 5.5 | 17.14 |
| 6.0 | 16.48 |
| 6.5 | 15.21 |
| 7.0 | 14.33 |

Use Simpson's Rule to estimate

$$\int_3^7 f(x)\,dx$$

$\Delta x = .5$




$N = 8$