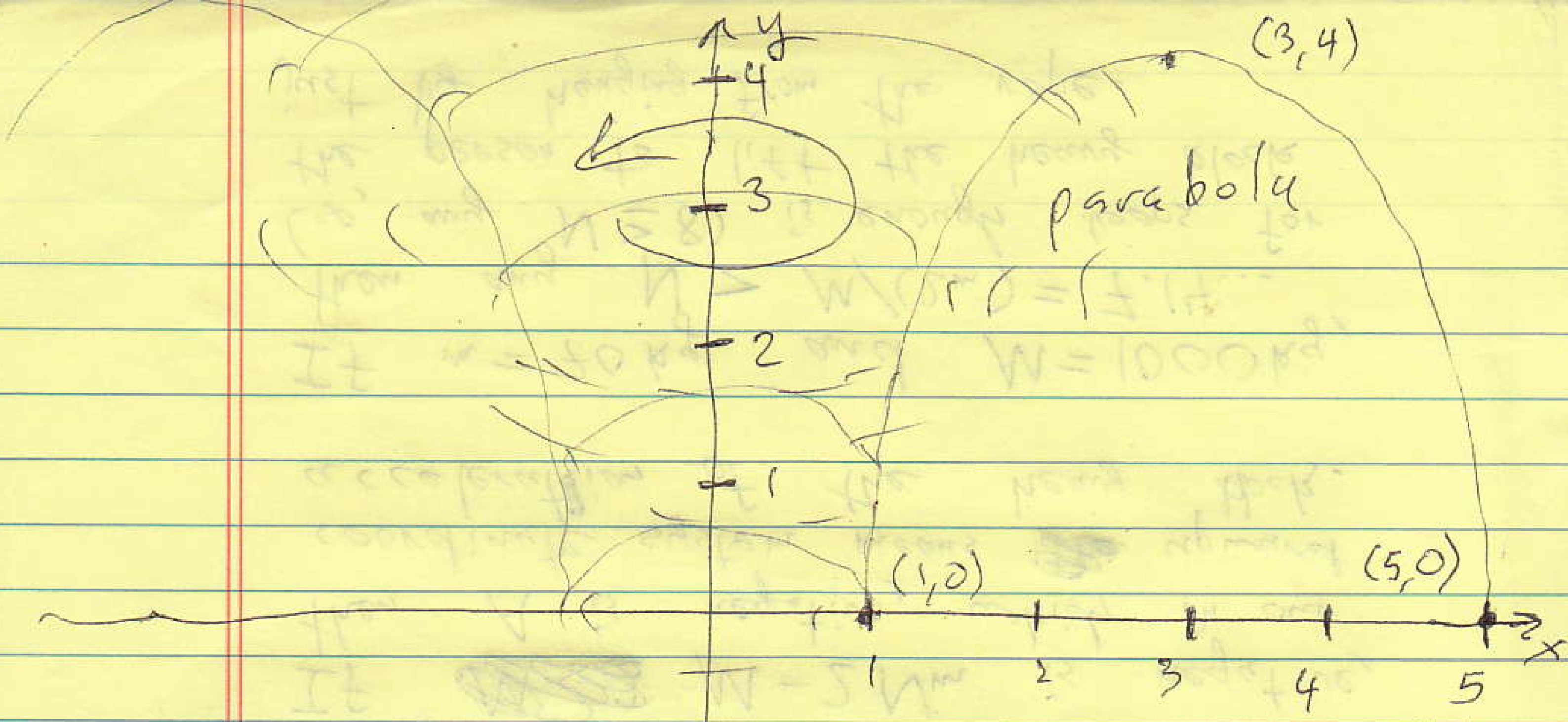


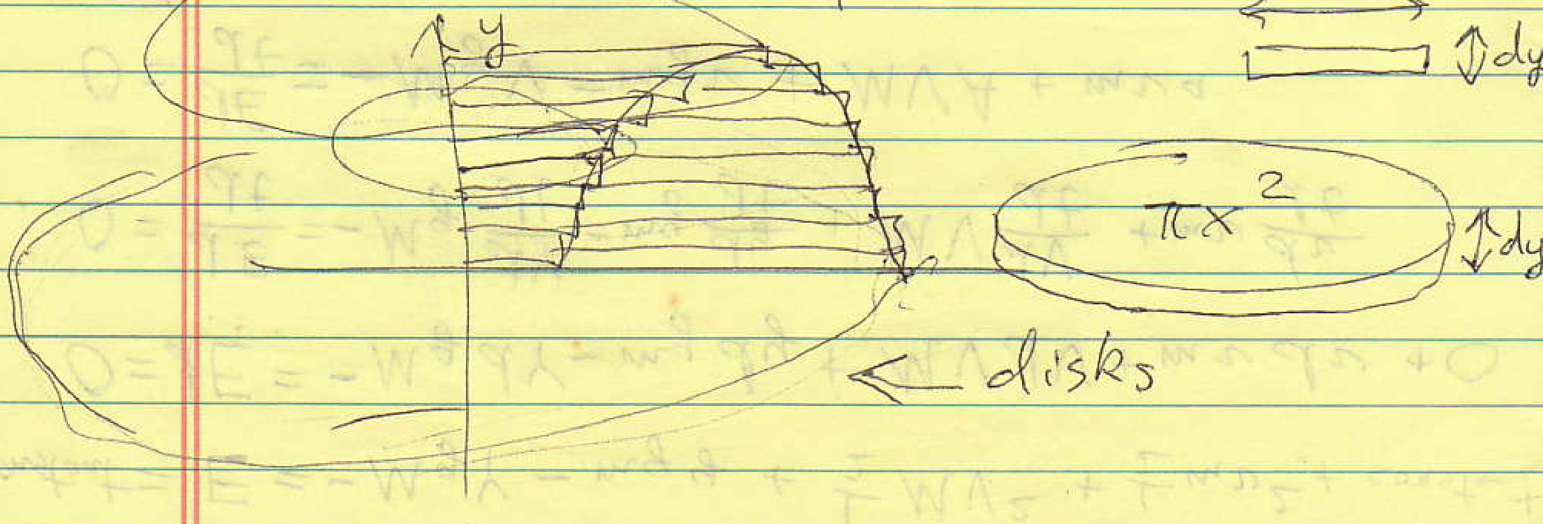
$y = 4 - 1(x-3)^2$

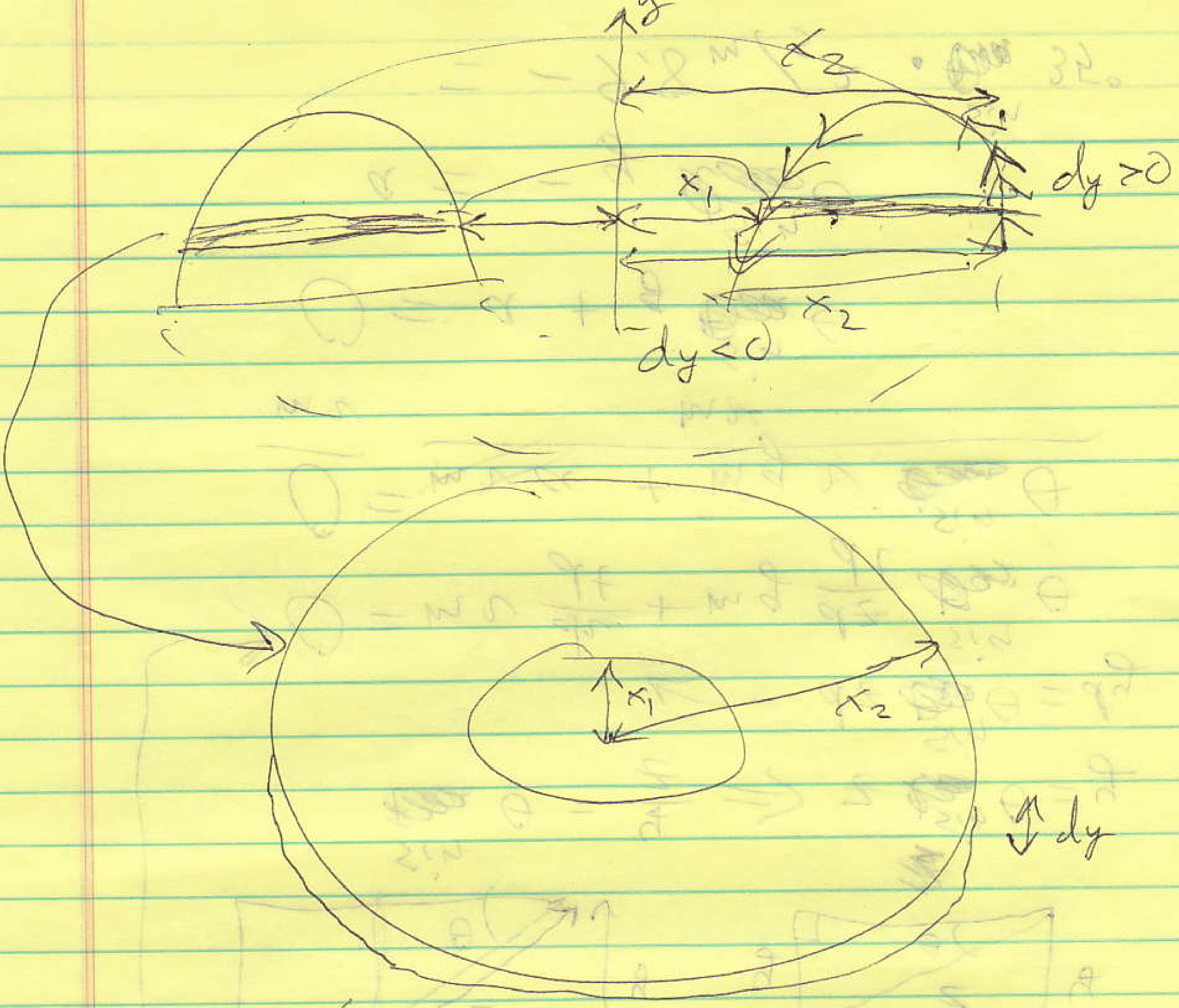
Works for $(1,0)$, $(3,4)$, $(5,0)$.

Volume of bundt cake = ?

Rotating $y = 4 - (x-3)^2$ around the y-axis.

Method 1: $\pm \int_{loop} \pi x^2 \, dy$



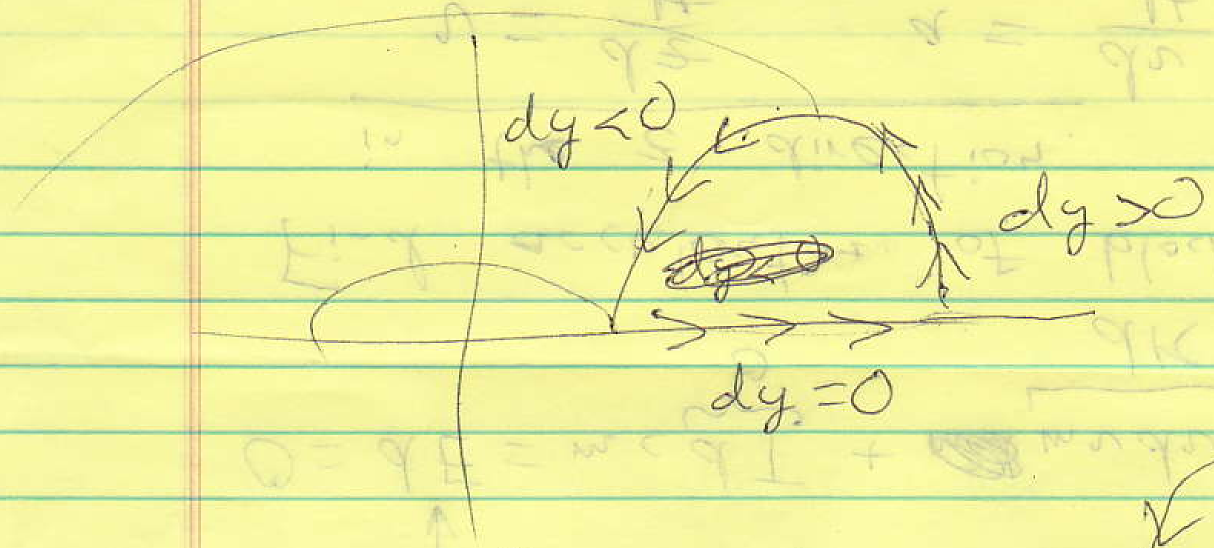


$$dV = \left(\pi x_2^2 - \pi x_1^2\right) dy$$

To get $V = \int_{\text{loop}} \pi x^2 \, dy$,

you need $dy < 0$ at x_1

and $dy > 0$ at x_2



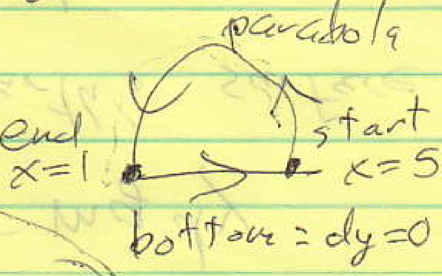
$V = \int_{loop} \pi x^2 \, dy$

Idea: inner "negative" disks & outer "positive" disks combine for the correct volume.

Recall that for revolving clockwise loops around x-axis, $V = \int_{loop} \pi y^2 \, dx$

Back to the cake:

$V = \int_{loop} \pi x^2 \, dy$



$V = \int_{parabola} \pi x^2 \, dy + \int_{bottom} \pi x^2 \, dy \Leftarrow = 0$

$$V = \int_{\text{parabola}} \pi x^2 \, \underline{dy} = \int_{x=5}^{x=1} \pi x^2 (-2)(x-3)\,dx$$

$$y = 4 - (x-3)^2$$

$$dy = 0 - 2(x-3)\,\underbrace{d(x-3)}_{dx - 0}$$

$$dy = -2(x-3)\,dx$$

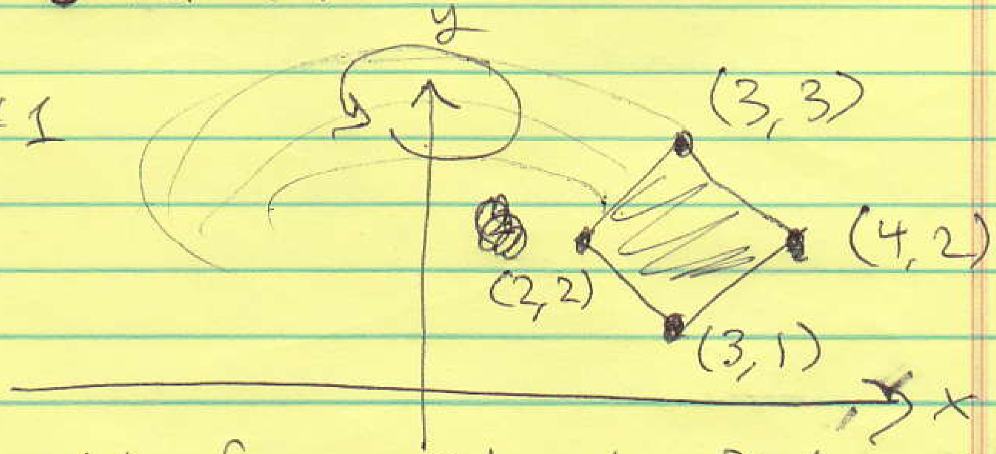
$$V = \int_5^1 -2\pi x^2 (x-3)\,dx = \int_1^5 2\pi x^2 (x-3)\,dx$$

$$= \int_1^5 2\pi (x^3 - 3x^2)\,dx = 2\pi \left(\frac{x^4}{4} - x^3\right)\Big|_1^5$$

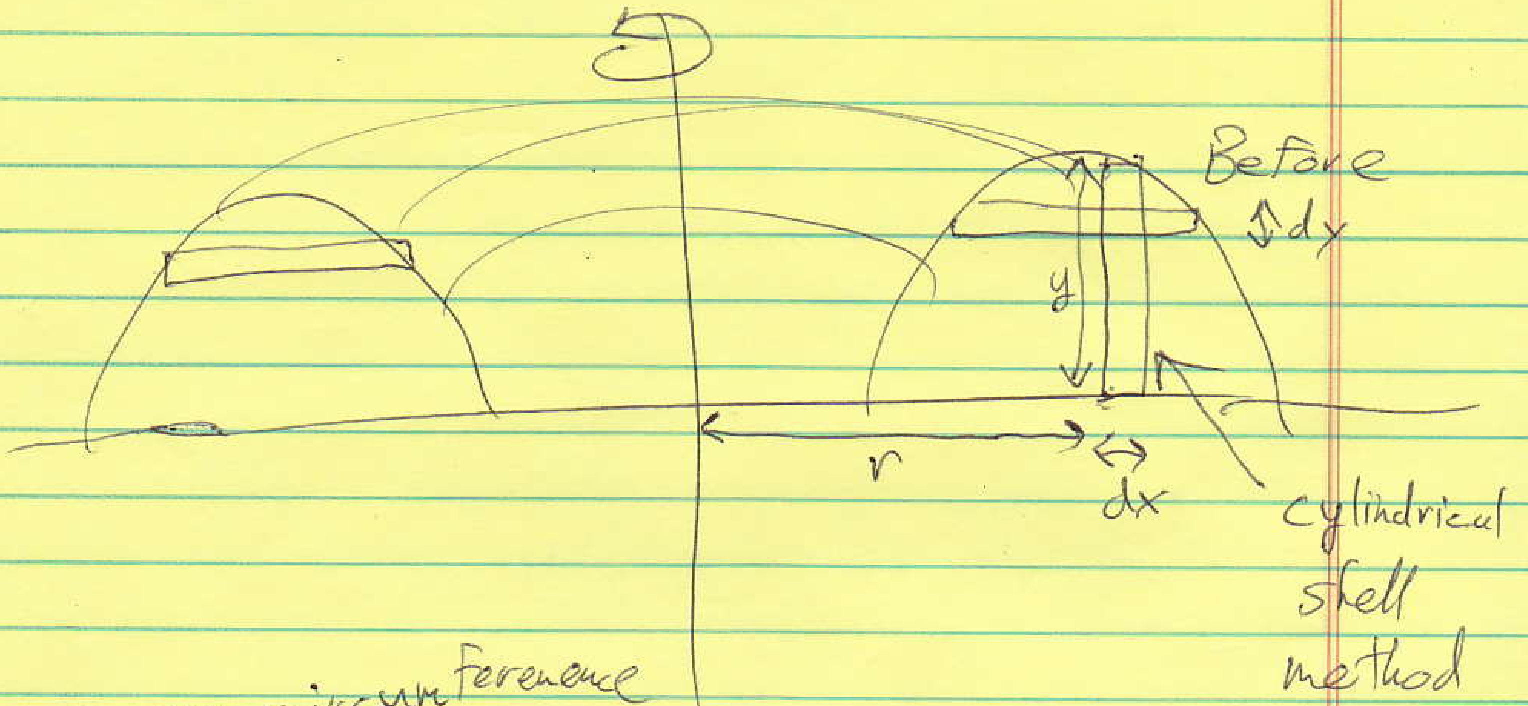
$$= \boxed{2\pi\left(\left(5^4/4 - 5^3\right) - \left(1^4/4 - 1^3\right)\right)}$$

$$= 64\pi \checkmark$$

HW #1

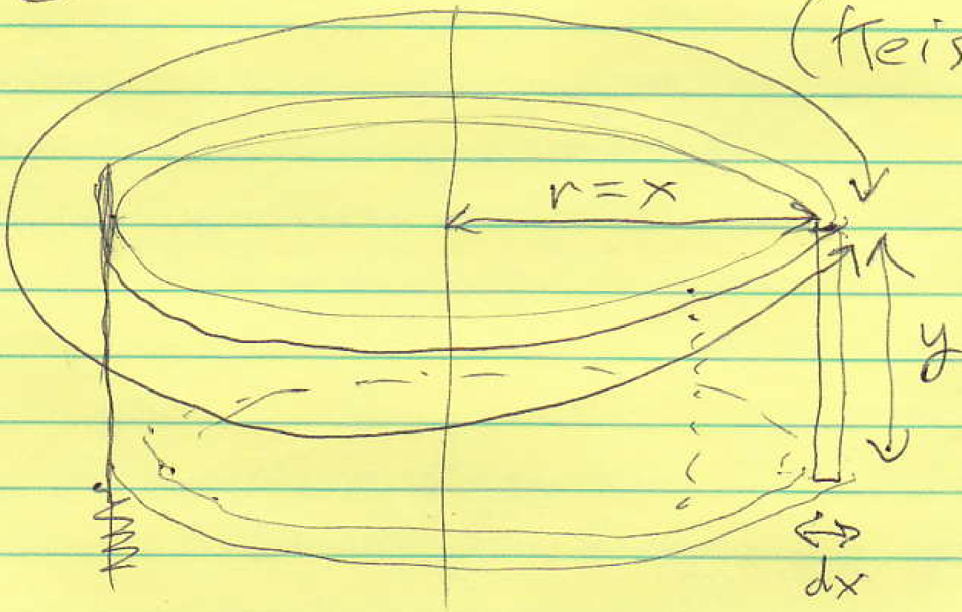


Use $V = \int_{\text{loop}} \pi x^2 \, dy$ to find the volume.



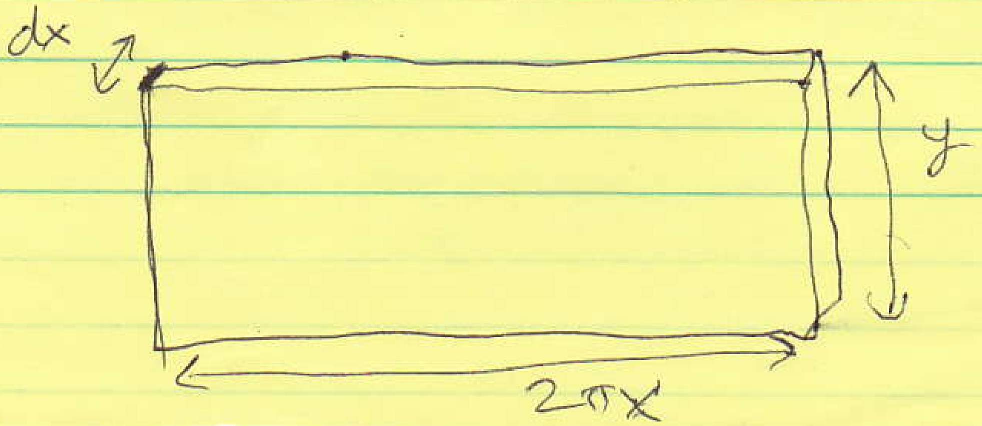
$2\pi r =$ circumference
$\parallel$
$2\pi x$

(Keisler, 6-2)

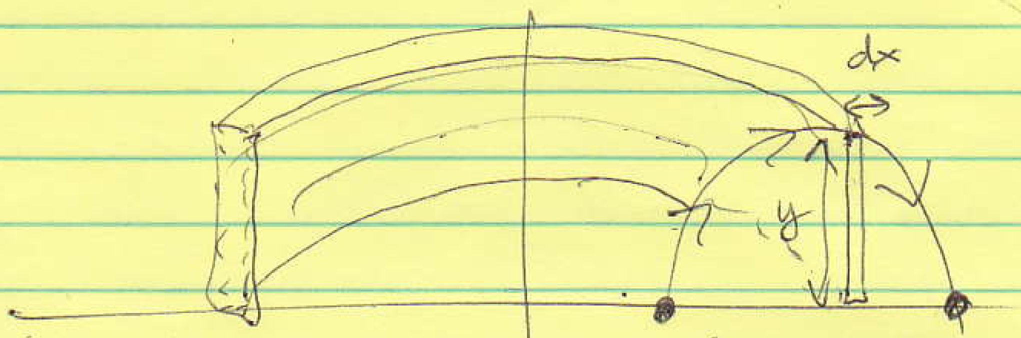


$$dV = 2\pi x \cdot y \cdot dx$$

Unroll the shell:



$$dV = 2\pi x\, y\, dx$$



$y = 4-(x-3)^2$ $\quad$ $x=1 \longleftrightarrow x=5$

x

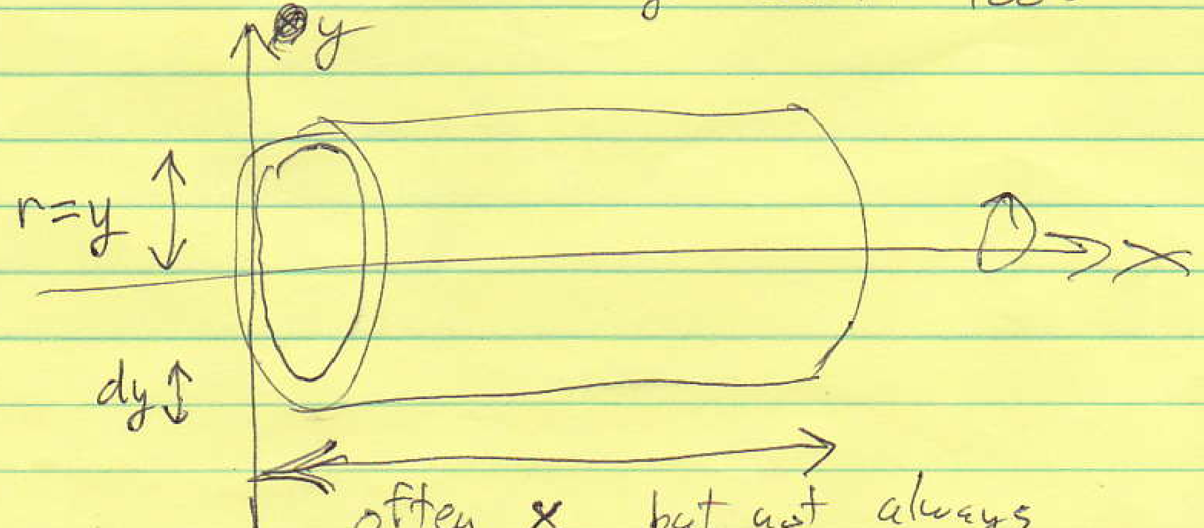
Want $dx \to 0$

$$V = \int_{x=1}^{x=5} 2\pi x\, y\, dx$$

$$V = \int_1^5 2\pi x \underbrace{\left(4-(x-3)^2\right)}_{y} dx$$

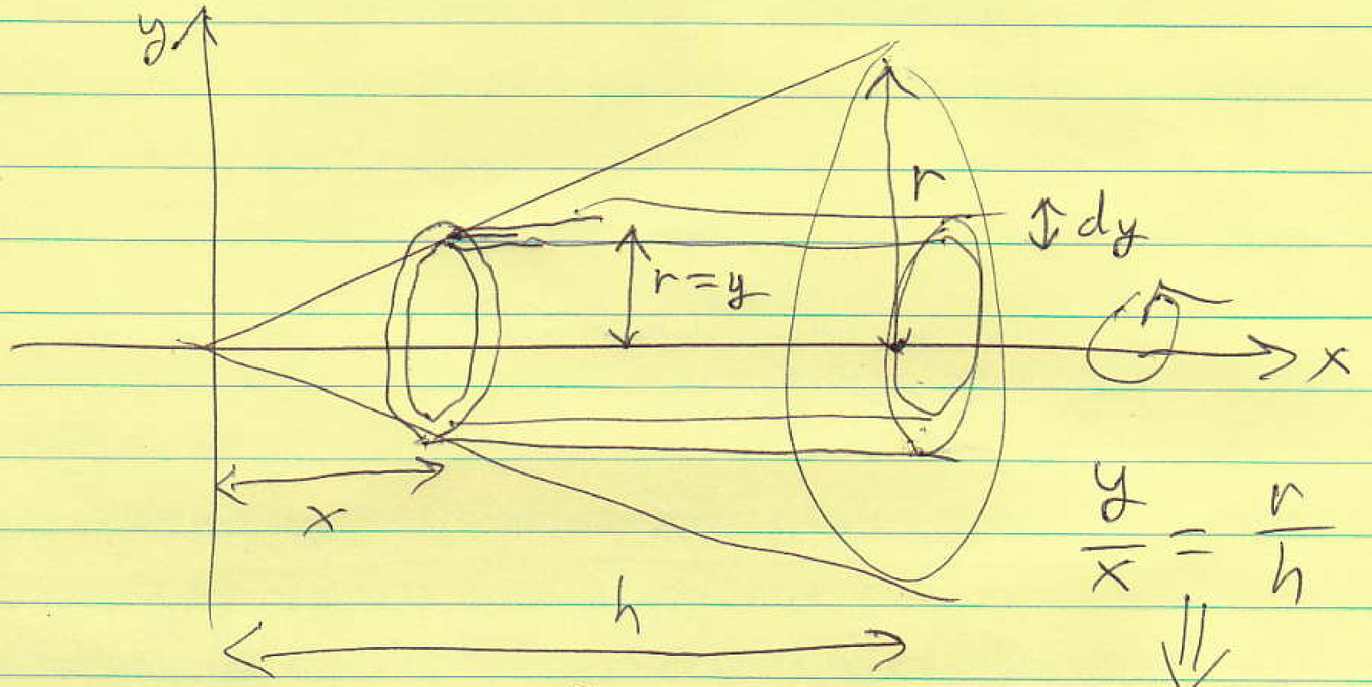
Should $= 64\pi$ too.

Shells this way work too:

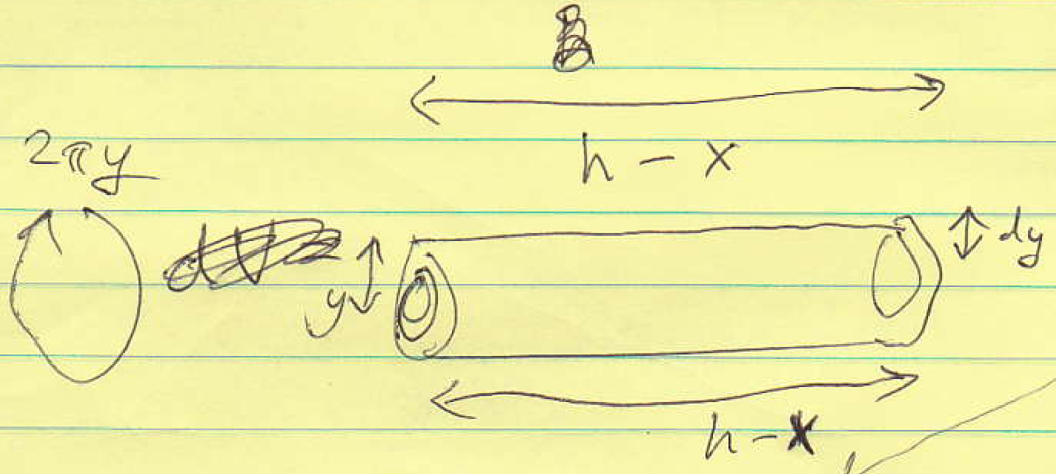


often x, but not always

Go back to cone example



$$\frac{y}{x} = \frac{r}{h}$$
$$\Downarrow$$
$$rx = yh$$
$$\Downarrow$$
$$x = \frac{yh}{r}$$



$$dV = 2\pi y\,(h - x)\,dy$$

$$V = \int_{0=y}^{r=y} 2\pi y\,(h - yh/r)\,dy$$

↳ Should $= \frac{1}{3}\pi r^2 h$ too.

HW #2 Use the cylindrical shell method to express the volume of the following donut as an integual with just one variable $(x, t, \text{or } y)$: