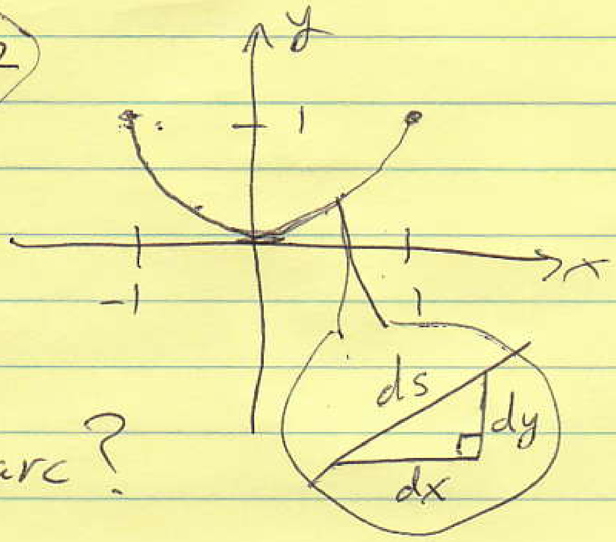
(notes OK)

Test #4 on March 10 (no calc)

Parabola $y = x^2$

$-1 \le x \le 1$



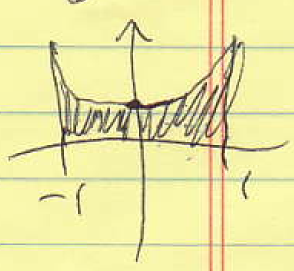
What is the length of this arc?

$$L = \int_{x=-1}^{x=1} ds = \int_{x=-1}^{x=1} \sqrt{dx^2 + dy^2}$$

$$y = x^2 \Rightarrow dy = 2x\,dx$$

$$L = \int_{x=-1}^{x=1} \sqrt{\underbrace{dx^2}_{1\,dx^2} + \underbrace{(2x\,dx)^2}_{4x^2dx^2}}$$

$y = \sqrt{1+4x^2}$



$$L = \int_{-1}^{1} \sqrt{1 + 4x^2}\, dx$$

even function

$$\sqrt{1+4(-x)^2} = \sqrt{1+4x^2}$$

$$L = 2\int_0^1 \sqrt{1+4x^2}\, dx$$

Idea $\sqrt{1+\tan^2\theta} = \underbrace{\sqrt{\sec^2\theta}}_{\pm\sec\theta}$

eliminate the $\sqrt{}$

Want $4x^2 = \tan^2\theta$

$$x^2 = \frac{\tan^2\theta}{4}$$

$$x = \pm\frac{\tan\theta}{2}$$

Pick $x = \frac{\tan\theta}{2}$

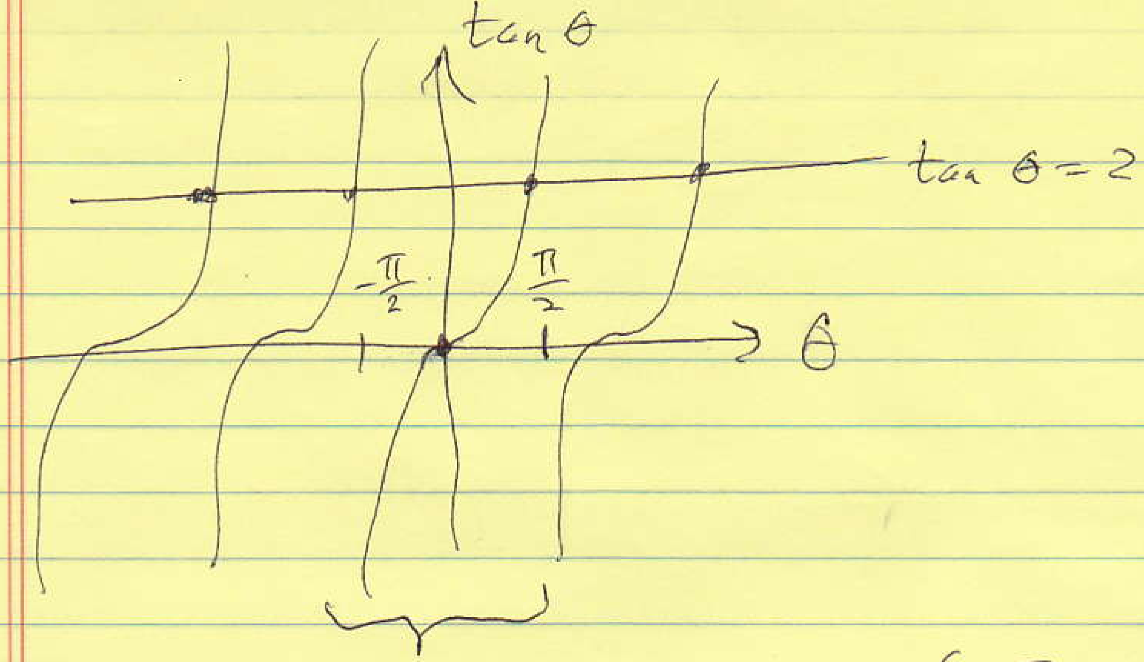
$$dx = \frac{\sec^2\theta \, d\theta}{2}$$

$$L = 2\int_0^1 \sqrt{1+4x^2}\,dx = 2\int_{x=0}^{x=1} \sqrt{\sec^2\theta}\ \frac{\sec^2\theta\,d\theta}{2}$$

$$= 2\int_0^{x=1} \pm\sec\theta \cdot \sec^2\theta \, d\theta/2$$

$$= \int_{x=0}^{x=1} \pm \sec^3\theta \, d\theta$$

$$x=1 \Rightarrow \frac{\tan\theta}{2} = 1 \Rightarrow \tan\theta = 2$$



Choose θ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1} 2 = \arctan 2 =$ the unique θ

between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$ satisfying

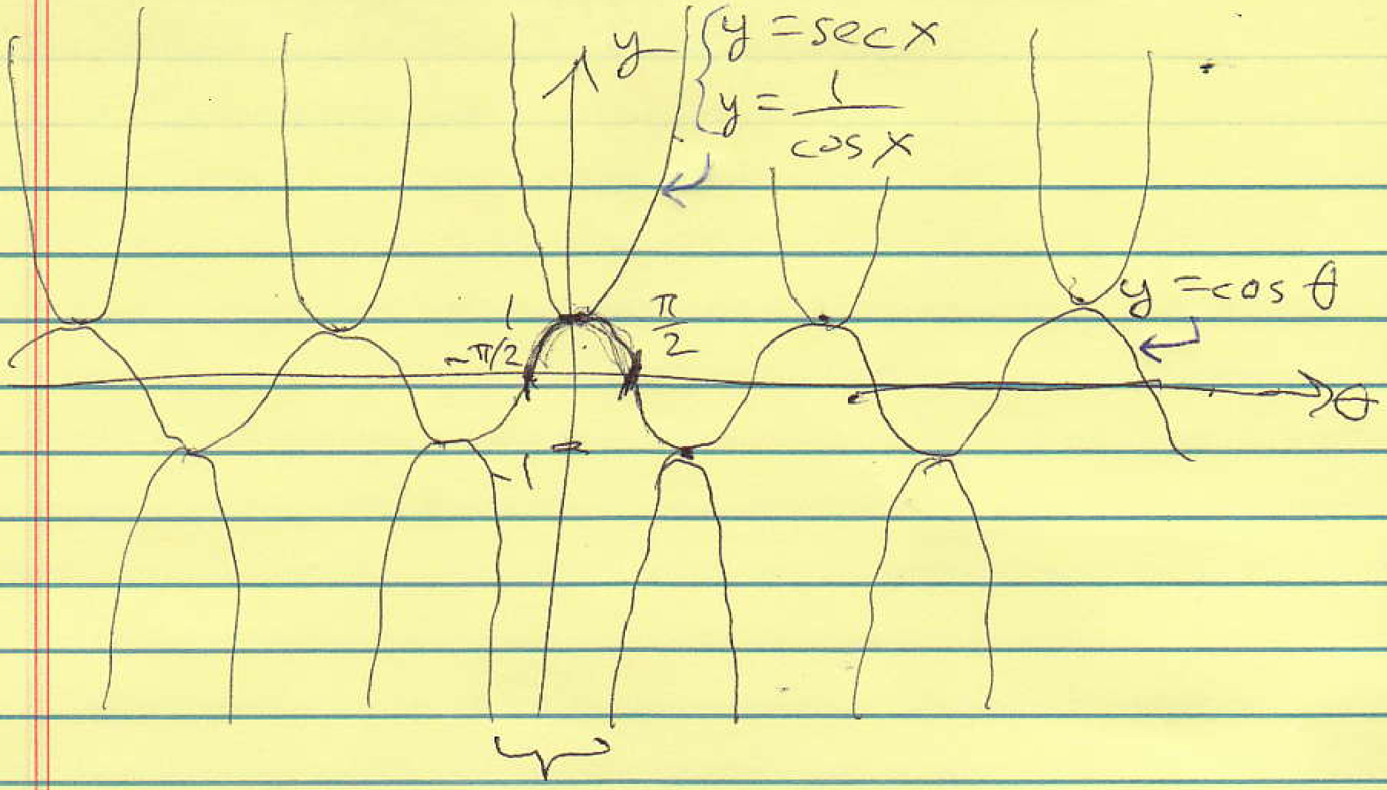
$\tan\theta = 2$

$x = 1 \Rightarrow \tan\theta = 2$ Pick $\theta = \arctan 2$

$x = 0 \Rightarrow \frac{\tan\theta}{2} = 0 \Rightarrow \tan\theta = 0$

Pick $\theta = \arctan 0$
$= 0$

$$L = \int_{\theta=0}^{\theta=\arctan 2} \pm \sec^3\theta \, d\theta$$



$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\sec\theta > 0$$

$$\sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta} = \sec\theta$$

$$L = \int_0^{\arctan(2)} \sec^3\theta \, d\theta$$

$$= \left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln\left|\sec\theta + \tan\theta\right|\right)\Bigg|_0^{\arctan(2)}$$

$$\sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1$$

$$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$$

$$\tan(\arctan(2)) = 2$$

$\sec(\arctan 2) = ?$

$1 + \tan^2(\arctan 2) = \sec^2(\arctan 2)$

$\sqrt{1+\tan^2(\arctan 2)} = \sec(\underbrace{\arctan 2}_{\uparrow})$

$\sqrt{1+(\tan(\arctan 2))^2}$ $\quad -\frac{\pi}{2} < \arctan(2) < \frac{\pi}{2}$

$\sqrt{1+2^2} = \sqrt{5}$ $\quad 0 < \sec(\arctan 2)$

$L = \frac{1}{2} \sqrt{5} \cdot 2 + \frac{1}{2} \ln|\sqrt{5}+2|$

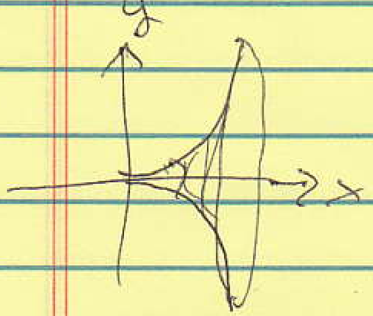
$- \left(\underbrace{\frac{1}{2} \cdot 1 \cdot 0}_{0} + \frac{1}{2} \ln \underbrace{|1+0|}_{1} \right)$

$\ln 1 = 0$

$L = \sqrt{5} + \frac{1}{2} \ln (\underbrace{\sqrt{5}+2}_{\sqrt{5}+2>0})$

so $|\sqrt{5}+2| = \sqrt{5}+2$

HW: Consider $y = x^2$ from $x = 0$ to $x = 1$.



Revolve that curve around the x-axis.

Find an exact formula for the surface area.