HW #1 Find an implicit solution to:

$$\begin{cases} \dfrac{dx}{dt} = 3x(4x-1)(x-2) \\ x = 5 \quad \text{when} \quad t=0 \end{cases}$$

and sketch its graph.

$$\int \frac{x^2\,dx}{(x-1)(x-2)(2x-3)} = \int \frac{x^2\,dx}{2x^3 + \_x^2 + \_x + \_}$$

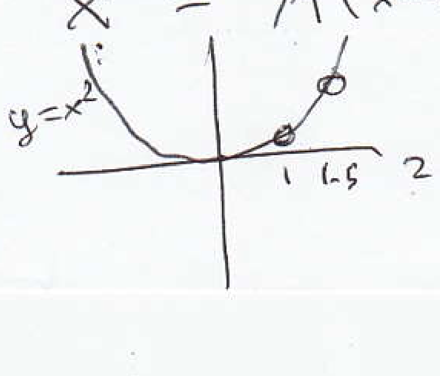
The denominator is a cubic (degree 3)

The numerator has degree $2 < 3$.

If deg(top) < deg(bottom) and you can factor the bottom, then you can use partial fractions.

$$\frac{x^2}{(x-1)(x-2)(2x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{2x-3}$$

$$x^2 = A(x-2)(2x-3) + B(x-1)(2x-3) + C(x-1)(x-2)$$



↳ This equation is true at <u>all</u> x, including 1, 1.5, 2.

$\cancel{x=1 \Rightarrow 1^2 = A(1-8)(1-2) + B\cdot 0 + C \cdot 0}$

$\cancel{1 = A(-1)(-2) = 2A}$

$x = 1 \Rightarrow 1^2 = A(1-2)(2\cdot 1-3) = A(-1)(-1) = A$

$x = 1 \Rightarrow 1 = A$

$x = 2 \Rightarrow A\cdot 0 + B(2-1)(2\cdot 2-3) + C\cdot 0 = 2^2$

$B(+1)(+1) = 4 \Rightarrow B = 4$

$x = 3/2 \Rightarrow$

$(3/2)^2 = A\cdot 0 + B\cdot 0 + C\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)$

$9/4 = C(1/2)(-1/2) = -C/4 \Rightarrow C = -9$

$$\int \frac{x^2\,dx}{(x-1)(x-2)(2x-3)} = \int \frac{1\,dx}{x-1} + \int \frac{4\,dx}{x-2} + \int \frac{-9\,dx}{2x-3}$$

$$\int \frac{dx}{x-1} = \int \frac{du}{u} = \ln|u| + c = \ln|x-1| + c$$

$u = x-1 \quad du = dx$

$$\int \frac{4\,dx}{x-2} = \int \frac{4\,dv}{v} = 4\ln|v| + c = 4\ln|x-2| + c$$

$v = x-2 \quad dv = dx$

$+c$

$$\int \frac{-9\,dx}{2x-3} = \int \frac{(-9/2)\,dw}{w} = -\frac{9}{2}\ln|w| = -\frac{9}{2}\ln|2x-3|$$

$w = 2x-3 \quad dw = 2dx \quad dx = dw/2$

HW #2 $\displaystyle\int_4^5 \frac{dx}{(2x-1)(x-7)} = ?$

If you have repeated factors:

$$\frac{x+2}{(x-1)^2(3x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{3x+1}$$

$$x+2 = A(x-1)(3x+1) + B(3x+1) + C(x-1)^2$$

$$\frac{x^3+7}{(x-1)^3(2x+6)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{2x+6}$$

Why? $\displaystyle\frac{x+2}{(x-1)^2(3x+1)} = \frac{A(x-1)+B}{(x-1)^2} + \frac{C}{3x+1}$

$$\frac{x+2}{(x-1)^2(3x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{3x+1}$$

$x+2 = A(x-1)(3x+1) + B(3x+1) + C(x-1)^2$

true for all x. Plug in $x=1$, $x=-\frac{1}{3}$

$x-1=0$ $\quad$ $3x+1=0$

Randomly pick an easy other x-value: $0=x$.

Do these first

$x=1 \Rightarrow 1+2 = 0 + B(3\cdot 1+1) + 0 = 4B$

$3 = 4B$ $\quad$ $\boxed{B = 3/4}$

$x = -\frac{1}{3} \Rightarrow -\frac{1}{3}+2 = 0 + 0 + C\left(-\frac{1}{3}-1\right)^2$

$-\frac{1}{3}+\frac{6}{3} = C\left(-\frac{1}{3}-\frac{3}{3}\right)^2 = C\left(-\frac{4}{3}\right)^2$

$\frac{5}{3} = C\left(\frac{16}{9}\right)$ $\quad$ $15 = 16C$

$\boxed{15/16 = C}$

$0+2 = A(0-1)(3\cdot 0+1) + B(3\cdot 0+1) + C(0-1)^2$

$2 = A(-1)(+1) + \frac{3}{4}(+1) + \frac{15}{16}(+1)$

$2 = -A + \frac{3}{4} + \frac{15}{16}$ $\quad$ $2 - \frac{3}{4} - \frac{15}{16} = -A$

$-2 + \frac{3}{4} + \frac{15}{16} = A$ $\quad$ $-\frac{32}{16} + \frac{12}{16} + \frac{15}{16} = A$

$\boxed{-\frac{5}{16} = A}$

$$\frac{x+2}{(x-1)^2(3x+1)} = \frac{-5/16}{x-1} + \frac{3/4}{(x-1)^2} + \frac{15/16}{3x+1}$$

HW #3 $\displaystyle\int \frac{x\,dx}{(2x+3)^2(1-x)} = ?$

What about quadratic factors that can't be factored into linear factors?

$x^2+4 = (\_x+\_)(\_x+\_)$?

Not without complex #s.

$$\frac{1}{(x^2+4)(x-3)} = \frac{Ax+B}{x^2+4} + \frac{C}{x-3}$$

$$1 = (Ax+B)(x-3) + C(x^2+4)$$

$$x=3 \Rightarrow \quad 1 = (3A+B)\underbrace{(3-3)}_{0} + C(3^2+4)$$

$$1 = C(13) \qquad C = 1/13$$

To find A, B, plug in 2 random #s for x.