3/21 Monday notes

$$\int \frac{dx}{x(M-x)} = \int k\,dt \Leftarrow \begin{cases} \frac{dx}{dt} = kx(M-x) \\ 0 < x < M \end{cases}$$

See

$$\frac{1}{M}\ln x - \frac{1}{M}\ln(M-x) = kt + c$$

$$\Rightarrow \frac{1}{M}\ln\frac{x}{M-x} = kt + c$$

t	x
t_0	x_0
t_1	x_1
t_2	x_2

t	x
0	15000
1	17000
3	18000

$$\ln\left(\frac{x_1}{M-x_1}\right) - \ln\left(\frac{x_0}{M-x_0}\right)$$

$$\frac{1}{M}\ln\left(\frac{x_1(M-x_0)}{x_0(M-x_1)}\right) = k(t_1-t_0) \rightarrow \text{solve for } k, \text{ given } M$$

$$\frac{1}{M}\ln\frac{x_0}{M-x_0} - kt_0 = c \rightarrow \text{solve for } c, \text{ given } M$$

$$\frac{1}{M}\ln\frac{x_2(M-x_0)}{x_0(M-x_2)} = k(t_2-t_0) = \frac{t_2-t_0}{t_1-t_0}\cdot\frac{1}{M}\ln\frac{x_1(M-x_0)}{x_0(M-x_1)}$$

$$\frac{x_2(M-x_0)}{x_0(M-x_2)} = \left(\frac{x_1(M-x_0)}{x_0(M-x_1)}\right)^{\left(\frac{t_2-t_0}{t_1-t_0}\right)}$$

$$\frac{18(M-15000)}{15(M-18000)} = \left(\frac{17(M-15000)}{15(M-17000)}\right)^3$$

$$18(M-15000)(15^3)(M-17000)^3 = 15(M-18000)(17^3)(M-15000)^3$$

$$\Rightarrow M \approx 18{,}118 \Rightarrow k = \frac{1}{18118}\ln\frac{17\cdot 3118}{15\cdot 1118} \approx 6.4\times10^{-5}$$

$$\Rightarrow c \approx 8.7\times10^{-5} \Rightarrow \text{when } t=5,$$

$$x = \frac{M}{1+e^{-M(kt+c)}} \approx \boxed{18{,}106} \text{ bacteria @ 5PM}$$