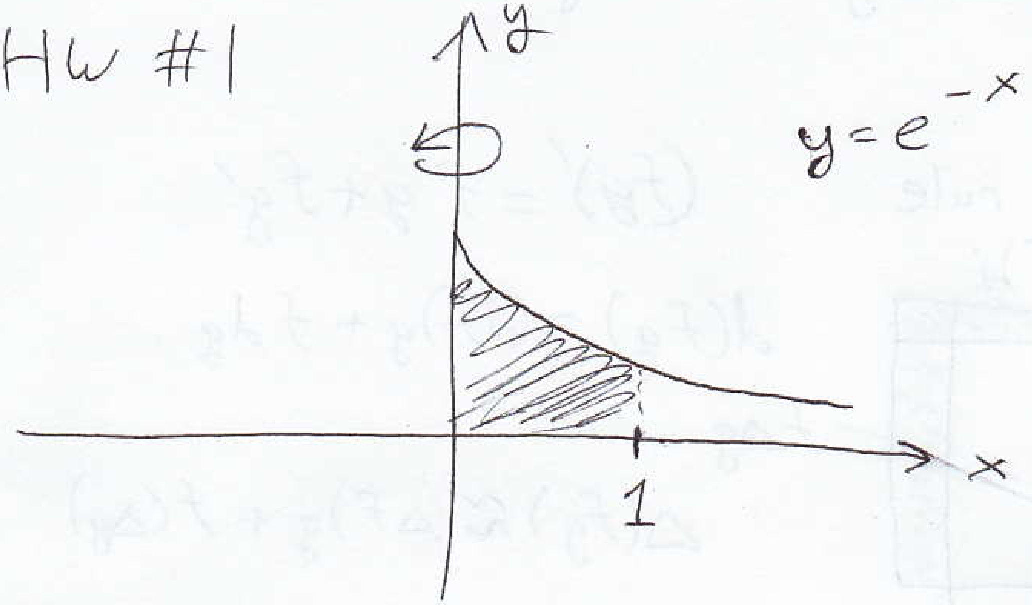
IBP continued

HW #1



Find the volume from revolving the region around the y-axis.

$$I=\int \sec^3\theta\, d\theta = \int \sec\theta \underbrace{\sec^2\theta}_{1+\tan^2\theta} d\theta$$

$$I = \underbrace{\int \sec\theta\, d\theta}_{\ln|\sec\theta+\tan\theta|+c} + \underbrace{\int \sec\theta \tan^2\theta\, d\theta}_{\int \underbrace{(\tan\theta)}_{u}\underbrace{(\sec\theta\tan\theta\, d\theta)}_{dv}}$$

$du = \sec^2\theta\, d\theta$

$v = \sec\theta$

$I = \ln|\sec\theta+\tan\theta| + c + uv - \int v\, du$ (IBP)

$$I = \ln|\sec\theta+\tan\theta| + c + \tan\theta\sec\theta - \underbrace{\underbrace{\int \sec\theta\sec^2\theta\,d\theta}_{\int \sec^3\theta\,d\theta}}_{I}$$

$$I = \ln|\sec\theta+\tan\theta| + \tan\theta\sec\theta - I + c$$

$$2I = \ln|\sec\theta+\tan\theta| + \tan\theta\sec\theta + c$$

$$I = \frac{1}{2}\left(\ln|\sec\theta+\tan\theta| + \tan\theta\sec\theta\right) + \frac{c}{2}$$

$$\int \sec^3\theta\,d\theta = I = \frac{1}{2}\ln|\sec\theta+\tan\theta| + \frac{1}{2}\sec\theta\tan\theta + c$$

Rename $\frac{c}{2}$ as c

It's arbitrary.

HW#2 Find a formula for

$$I = \int e^{ax}\cos(bx)\,dx.$$

Use IBP twice to get an equation you can solve for I.

① $\frac{dx}{dt} + 5x = 3$ ② $\frac{dx}{dt} = -5x + 3$

③ $dx = (-5x + 3)dt$ ④ $\frac{dx}{-5x+3} = dt$

$$\underbrace{\int \frac{dx}{-5x+3}}_{\substack{u = -5x+3 \\ du = -5dx \\ -\frac{1}{5}du = dx}} = \int dt = t + c$$

$$= \int \frac{(-1/5)\,du}{u}$$

$$\parallel$$

$$-\frac{1}{5}\ln|u| \underbrace{}_{+c}$$

put on other side of equation with t+c.

$$-\frac{1}{5}\ln|u| = t + c$$

$$\ln|u| = -5(t+c)$$

$$|u| = e^{-5(t+c)}$$

$$u = \pm e^{-5(t+c)} = \underbrace{\left(\pm e^{-5c}\right)}_{\text{call this } A} e^{-5t}$$

$$-5x + 3 = u = Ae^{-5t}$$

$$3 - Ae^{-5t} = 5x \Rightarrow x = \underbrace{\frac{3 - Ae^{-5t}}{5}}_{\text{general solution to } \frac{dx}{dt} + 5x = 3}$$

$$\frac{dx}{dt} + \underbrace{5t^2}_{\frac{dp}{dt}} x = \underbrace{3}_{q}$$

Need a new technique

General method: p, q functions of t only

$$\frac{dx}{dt} + \frac{dp}{dt}x = q$$

$$e^p\left(\frac{dx}{dt} + \frac{dp}{dt}x\right) = e^p q$$

$$\frac{d}{dt}\left(e^p x\right) = \frac{d}{dt}(e^p)x + e^p\frac{dx}{dt}$$

product rule

$$\frac{d(e^p)}{dt} = \frac{e^p\,dp}{dt} = e^p\frac{dp}{dt}$$

$$\frac{dp}{dt} = 5t^2$$

$$\int dp = \int 5t^2\,dt$$

$$p = \frac{5}{3}t^3$$

choose simplest antiderivative

$$\frac{d}{dt}(e^p x) = e^p \frac{dp}{dt} x + e^p \frac{dx}{dt}$$

$$= e^p\left(\frac{dx}{dt} + \frac{dp}{dt}x\right) = e^p q$$

We had $\frac{dx}{dt} + \frac{dp}{dt}x = q.$

Now we have $\frac{d}{dt}(e^p x) = e^p q$

$\int d(e^p x) = \int e^p q \, dt \qquad e^p x = \int e^p q \, dt$

$p = \frac{5}{3}t^3 \qquad e^{\frac{5}{3}t^3} x = \underbrace{\int e^{\frac{5}{3}t^3} 3 \, dt}_{\psi(t) + c}$

$q = 3$

$$x = e^{-\frac{5}{3}t^3}\left(\psi(t) + c\right)$$

Concretely, $\psi(t) = \int_0^t e^{\frac{5}{3}t^3} 3 \, dt$

$$t\frac{dx}{dt}+x=2t$$

$$\frac{dx}{dt}+\frac{x}{t}=2$$

$$\frac{dx}{dt}+\underbrace{\frac{1}{t}}_{\frac{dp}{dt}}x=\underbrace{2}_{q}$$

$$p=\ln|t| \Rightarrow e^{p}=e^{\ln|t|}=|t|$$

$$\frac{d}{dt}(|t|x)=\frac{d}{dt}(e^{p}x)=e^{p}q=2|t|$$

Cases $\begin{cases} t\geq 0: & |t|=t \\ t<0: & |t|=-t \end{cases}$

Case $t\geq 0$: $\frac{d}{dt}(tx)=2t$

Case $t<0$: $\frac{d}{dt}(-tx)=-2t$

Both same as $\frac{d}{dt}(tx)=2t$

$$\int d(tx) = \int 2t\,dt \Rightarrow tx = t^2 + c$$

$$x = t + \frac{c}{t}$$

We can check that

$$t\frac{dx}{dt} + x = 2t \ldots$$