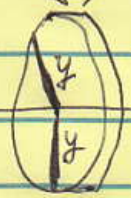
Revolved around x-axis

clockwise loop

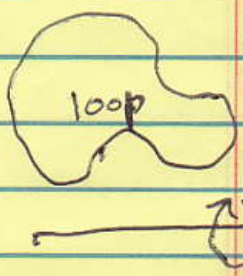
$$V = +\int_{\text{loop}} \pi y^2 \, dx$$

$\pm dx$



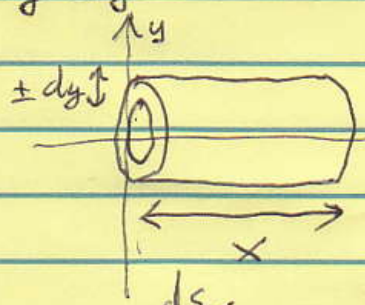
counterclockwise loop

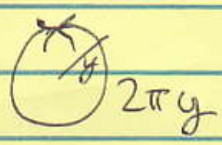
$$V = -\int_{\text{loop}} \pi y^2 \, dx$$



$$V = -\int_{\text{loop}} 2\pi x y \, dy$$

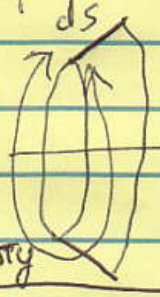
$$V = +\int_{\text{loop}} 2\pi x y \, dy$$





surface area:

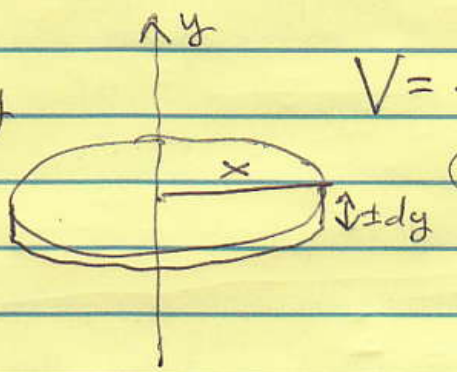
$$A = \int_{\text{loop}} 2\pi y \sqrt{dx^2 + dy^2}$$



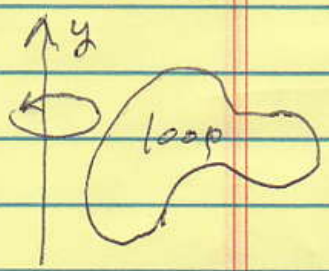
$$A = \int_{\text{loop}} 2\pi y \sqrt{dx^2 + dy^2}$$

Revolved around y-axis

$$V = -\int_{\text{loop}} \pi x^2 \, dy$$



$$V = +\int_{\text{loop}} \pi x^2 \, dy$$



$$V = +\int_{\text{loop}} 2\pi x y \, dx$$

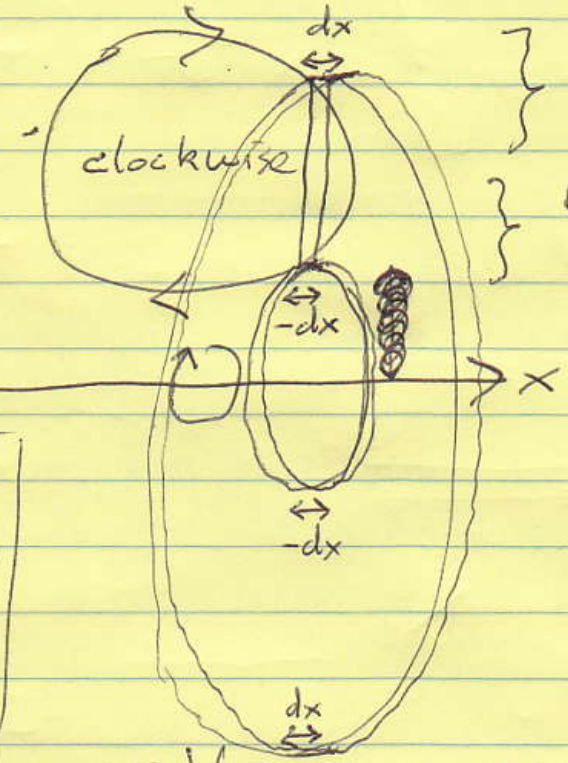
$$V = -\int_{\text{loop}} 2\pi x y \, dx$$

surface area:

$$A = \int_{\text{loop}} 2\pi x \sqrt{dx^2 + dy^2}$$

$$A = \int_{\text{loop}} 2\pi x \sqrt{dx^2 + dy^2}$$

Example justifications of sign:



top: going to right, so $dx > 0$, so thickness $= dx$

bottom: going to left, so $dx < 0$, so thickness $= -dx$

volume of clockwise loop revolved around x-axis

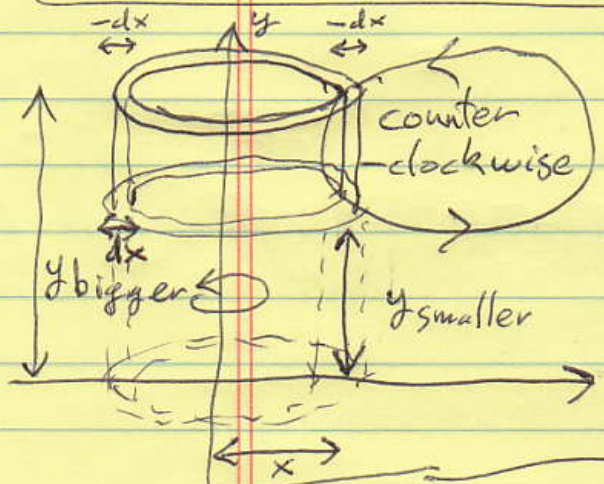
washer method for loops

You want to subtract the smaller disk from the bigger disk:

$$V_{washer} = \pi y_{bigger}^2 dx - \pi y_{smaller}^2 (-dx)$$

$$V_{washer} = +\pi y_{bigger}^2 dx + \pi y_{smaller}^2 dx$$

$$V = +\int_{loop} \pi y^2 dx$$



top: going to left, so $dx < 0$, so thickness $= -dx$

bottom: going to right, so $dx > 0$, so thickness $= dx$

$$V_{shell} = [2\pi y_{bigger} \times (-dx)] - [2\pi y_{smaller} \times dx]$$

$$V = -\int_{loop} 2\pi x y \, dx$$

volume of counter-clockwise loop revolved around y-axis: cylindrical shell method for loops