

Proof of area formula:

~~See separate int.~~

$$A = \underbrace{2b \Delta x}_{\text{rectangle}} + \int_{-\Delta x}^{\Delta x} f(t) dt \quad \text{where } f(0) = 0$$

See illustration below.

& $f(-\Delta x) = a - b$ & $f(\Delta x) = c - b$
 & f is quadratic

$$f(t) = pt^2 + qt \quad (\text{because } f(0) = 0)$$

$$(a+c-2b) = f(-\Delta x) + f(\Delta x) = 2p(\Delta x)^2$$

$$p = \frac{a+c-2b}{2(\Delta x)^2}$$

$$\int_{-\Delta x}^{\Delta x} (pt^2 + qt) dt = 2p \int_0^{\Delta x} t^2 dt + 0$$

even
odd
even part
odd part

$$= 2p \frac{\Delta x^3}{3} = \frac{a+c-2b}{3} \Delta x \quad (\text{Next: add } 2b\Delta x)$$

$$\Rightarrow A = \frac{a+4b+c}{3} \Delta x$$

~~add to area~~

