

1. [33 points] $\int \sin^2(5x) dx = ?$

$$u = 5x \quad du = 5dx$$
$$du/5 = dx$$

$$\int \sin^2 u \cdot \frac{du}{5}$$

$$\int \frac{1 - \cos 2u}{2} \cdot \frac{du}{5}$$

$$w = 2u \quad dw = 2du$$
$$dw/2 = du$$

$$\int \frac{1 - \cos w}{2} \cdot \frac{dw/2}{5}$$

$$\frac{1}{20} \int (1 - \cos w) dw$$

$$\frac{1}{20} (w - \sin w) + c$$

$$\frac{1}{20} (2u - \sin 2u) + c$$

$$\frac{1}{20} (10x - \sin(10x)) + c$$

2. [33 points] $\int_0^{\pi/3} \tan^3 x \sec x dx = ?$

$$\int_0^{\pi/3} \underbrace{\tan^2 x}_{u^2-1} \underbrace{\sec x \tan x dx}_{du}$$

$$u = \sec x$$

$$du = \sec x \tan x dx$$

$$\tan^2 x = \sec^2 x - 1 = u^2 - 1$$

$$x = \pi/3 \Rightarrow u = \sec \frac{\pi}{3} = \frac{1}{\cos \frac{\pi}{3}} = \frac{1}{1/2} = 2$$

$$x = 0 \Rightarrow u = \sec 0 = 1/\cos 0 = 1/1 = 1$$

$$\int_1^2 (u^2 - 1) du \longrightarrow = \left(\frac{u^3}{3} - u \right) \Big|_1^2$$

~~$$\int_1^2 (u^2 - 1) du = \left(\frac{u^3}{3} - u \right) \Big|_1^2$$~~

~~$$\left(\frac{2^3}{3} - 2 \right) - \left(\frac{1^3}{3} - 1 \right)$$~~

~~8-2-1~~

$$= \frac{8}{3} - \frac{6}{3} - \frac{1}{3} + \frac{3}{3} = \boxed{\frac{4}{3}}$$

TRIGONOMETRY

Angle Measurement

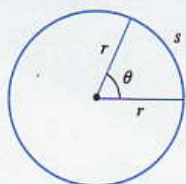
$$\pi \text{ radians} = 180^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$s = r\theta$$

(θ in radians)



Right Angle Trigonometry

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

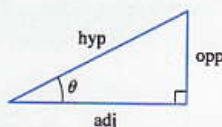
$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$



Trigonometric Functions

$$\sin \theta = \frac{y}{r}$$

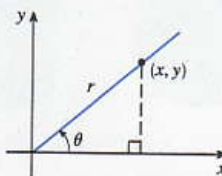
$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

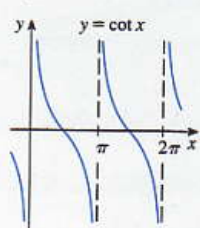
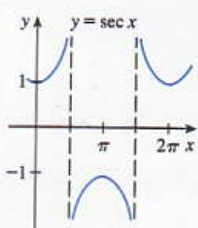
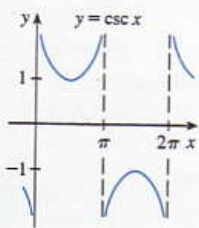
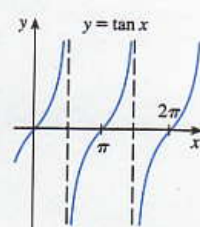
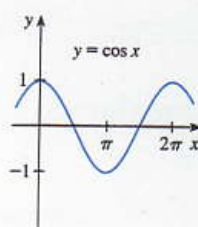
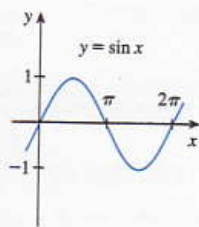
$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$



Graphs of Trigonometric Functions



Trigonometric Functions of Important Angles

θ	radians	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—

Fundamental Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

The Law of Sines

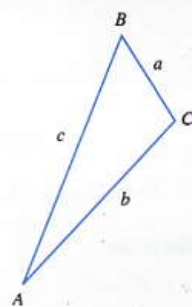
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Addition and Subtraction Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$