MATH 2415 Final Name: ______________________________

Try to solve at least four of the five test problems
and circle the four problems you want graded.
(I will grade the first four if you don't circle four.)

1. Find the three (interior) angles of the triangle with vertices $(0,0,0)$, $(1,2,3)$, and $(8,6,0)$.

2. Find the surface area of the ellipsoid parametrized as follows.

$$\begin{aligned} x &= 7\sin(u)\cos(v) & \qquad 0 \le u \le \pi \\ y &= \sin(u)\sin(v) & \qquad 0 \le v \le 2\pi \\ z &= 3\cos(u) & \end{aligned}$$

For full credit, you must (1) express the exact area as an integral whose only variables are u and v and (2) numerically estimate that integral (presumably using your calculator).

3. Find a function $f(x, y)$ with gradient

$$\langle P, Q\rangle = \langle 21x^2 + 5y^2 + 2y, 10xy + 2x + 12y^2\rangle,$$

or explain why no such function exists.

4. Find the flux of $\vec{F} = \langle x^2 + \cos(yz), \sin(zx) - y^2, z^2 \rangle$ through the (positively oriented) boundary of the solid tetrahedron

$$T = \{(x, y, z) \mid 0 \le x \le z \le y \le 5\}.$$

For full credit, you must (1) express the flux in terms of an integral (or integrals) and (2) evaluate the integral(s).

5. Find the point on the ellipse

$$E = \{(x, y) \mid (x/4)^2 + (y/3)^2 = 1\}$$

closest to the point $(2, 5)$ using the method of Lagrange multipliers.