MATH 2415 TEST 6

Name:

Exercise	Point Possible	Score
1	20	
2	40	
3	40	
Total	100	

1. [20 points] True or false? The integral
$$\int_0^{2\pi} \int_0^2 \int_r^2 dz \, dr \, d\theta$$
represents the volume enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$.
Explain why the statement is true, or explain why it is false.

2. [40 points] Consider the coordinate transformation T that sends (u, v) to $(x, y) = (kv^3, u/v^2)$ where k is a constant.
   (a) Compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.
   (b) Which, if any, values of k make T area-preserving?
       (By "area-preserving," I mean such that $dA = du \, dv$.)

3. [40 points] Laredo is at latitude $27.5°$ N, which is ϕ-coordinate $\frac{\pi}{2} - \frac{27.5\pi}{180}$ in spherical coordinates. The equator is at ϕ-coordinate $\pi/2$. Modeling the earth as a solid sphere, what percentage of the earth's volume has ϕ-coordinate between that of Laredo and that of the equator? (You do not need to know the radius of the earth to answer this question.)