MATH 2415 TEST 7

Name: ______________________________

Date: April 23, 2013.

Exercise	Point Possible	Score
1	35	
2	40	
3	25	
Total	100	

1. [35 points] Consider the vector field $\langle P, Q\rangle = \langle y, x-y\rangle$. Find a solution f to $\vec{\nabla} f = \langle P, Q\rangle$, if one exists. If there is no solution, then explain why not.

2. [**40 points**] A thin wire is bent into the shape of a semicircle $x^2 + y^2 = 4$, $x \geq 0$. If the linear density is a constant k, find the x-coordinate of the center of mass of the wire.

3. [25 points] Consider the following parametric curve C.

$$\begin{aligned} x(t) &= \cos(\pi t) \\ y(t) &= t - t^3 \\ -1 \leq & t \leq 1. \end{aligned}$$

The curve C is a loop (see next page) and it parametrizes the positively oriented boundary of the region D it encloses. Write a single integral equal to the area of D. You do not need to evaluate the integral, but you do need to eliminate all variables except t, so that a computer could evaulate the integral. (For example, if you believe $\int_C y^4\,dx$ equals the area of D, then your answer should be $\int_{-1}^{1} -\pi(t-t^3)^4 \sin(\pi t)\,dt$.)