MATH 2415 Final Exam Name: _____

Solve five of the six test problems. To indicate which you want graded, circle five of these numbers: 1 2 3 4 5 6.

1. Let H and K be the planes $x + y + z = 2$ and $5x = y + z$. Find a vector $\vec{w}$ that is parallel to both planes.

2. Consider the helix $\vec{r}(t) = \langle \cos(4t), 3t, \sin(4t) \rangle$. Find the unit normal vector at time $t = \pi/12$.

3. Suppose that $z = f(x, y)$, $x = g(t)$, $y = h(t)$, and, at time $t = 0$, we have $x = 0$, $y = 0$, $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = 2$, and $\frac{dz}{dt} = 3$, and, at position $(x, y) = (0, 0)$, we have $\frac{\partial f}{\partial x} = 4$. At position $(x, y) = (0, 0)$, what is $\frac{\partial f}{\partial y}$?

4. Find the center of mass of $\{(x, y) : 0 \leq r \leq 1 \text{ and } \pi/4 \leq \theta \leq \pi/3\}$. Assume density $dm/dA = 1$.

5. What fraction of the night sky can be seen from Laredo? To answer this question, find $A/(4\pi)$ where 4π is the surface area of the unit sphere and A is the surface area of

$$S = \{(x, y, z) : \rho = 1 \text{ and } 0 \leq \phi \leq 152.5^\circ\}.$$

6. What is the flux of $\vec{F} = \langle x^2, y^2, xyz \rangle$ through the disk

$$S = \{(x, y, z) : x = 1 \text{ and } (y-1)^2 + (z-1)^2 \leq 1^2\}?$$

Assume orientation $\vec{n} = \langle 1, 0, 0 \rangle$.