MATH 2415 Final Name: ______________________________

1. Let S be the flat triangular surface with vertices $A = (0, 0, 1)$, $B = (1, 0, 3)$, $C = (0, 2, 0)$ and orientation $\mathbf{N} = \langle 4, -1, 2\rangle/\sqrt{21}$. If the curl of $\mathbf{F}$ is $\langle 0, 3, 1\rangle$, then is the circulation of $\mathbf{F}$ along the boundary loop ∂S positive, negative, or zero?

2. Find the flux of $\mathbf{F} = \langle 2xy, 5yz, 8zx \rangle$ through the boundary surface of the solid cube

$$\{(x, y, z) \mid 2 \le x \le 3 \text{ and } 1 \le y \le 2 \text{ and } 0 \le z \le 1\}.$$

3. Assuming the Earth to be a sphere of radius of $R = 6370$ (kilometers), find the area of the part of the Earth's surface with latitudes between 0 (the equator) and $27.5\pi/180$ radians North (Laredo's latitude). (Warning: ϕ (or φ) is colatitude, not latitude.)

4. For the curve $\mathbf{r} = \langle 7/t,\ 5/t^2,\ 3t \rangle$, find the radius of curvature at $t = -1$.

5. Find the distance from $(3, 4, 5)$ to the line given by $\mathbf{r} = \langle 6t - 1, 4 - 2t, 1 - t\rangle$.

6. The function $f(x, y) = x^4 - xy + y^2$ has three critical points. One of them is a saddle point and the other two are locations of a local minimum.

(i) Compute the partial derivatives $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$.

(ii) Prove that $(0, 0)$ is a saddle point.

(iii) Extra credit: Find the other two critical points.