MATH 2415 Test 4   Name: _____

Note: all the integrals for this test can be done by hand, but maybe not as quickly and reliably as with a calculator.

1. Compute $\iint_S x \, dy \, dx$ where S is the interior of the triangle with vertices $(0,0)$, $(0,6)$, and $(5,0)$. (Using a coordinate transformation will work, but this triangle is nice enough that some of you may find it easier not to use a coordinate transformation.)

2. Assuming density $dm/dV = 10z$, express the mass in the region

$$\{(x,y,z) \mid 0 \le z \le e^{-r^2} \text{ and } 1 \le r \le 2\}$$

as a triple integral of the form $\int_\square^\square \int_\square^\square \int_\square^\square \square \, d\square \, d\square \, d\square$. Then evaluate your integral.

$$m = \int_0^{2\pi} \int_1^2 \int_0^{e^{-r^2}} 10z \, r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_1^2 5 r \, e^{-2r^2} \, dr \, d\theta = \int_0^{2\pi} \frac{5}{4}\left(e^{-2} - e^{-8}\right) d\theta = \frac{5\pi}{2}\left(e^{-2} - e^{-8}\right)$$