HW9

(1) $A, B, C \in$ plane P where
$A = (0,0,2)$, $B = (0,3,2)$, $C = (4,3,2)$

$P \parallel \overrightarrow{AB}, \overrightarrow{BC}$. $\quad P \perp \overrightarrow{AB} \times \overrightarrow{BC} = \vec{v}$

$\vec{v} = 3\vec{j} \times 4\vec{i} = -12\vec{k}$

$P = \{(x,y,z) \mid \underbrace{\langle x,y,z\rangle - \langle 0,0,2\rangle \perp \vec{v}}\}$

$\iff \langle x, y, z-2\rangle \bullet \langle 0,0,-12\rangle = 0$

$\iff (z-2)(-12) = 0$

$\iff \boxed{z = 2}$

(2) $L = P \cap Q$ where:

$$\begin{cases} P = \{(x,y,z) \mid 1x - 3y + 1z = 6\} \\ Q = \{(x,y,z) \mid 5(x-2) + 1y - 1z = 4\} \end{cases}$$

$P \perp \langle 1,-3,1\rangle$ & $Q \perp \langle 5,1,-1\rangle$.

So, $L \perp \langle 1,-3,1\rangle, \langle 5,1,-1\rangle$.

So, $L \parallel \langle 1,-3,1\rangle \times \langle 5,1,-1\rangle = \langle 2,6,16\rangle$

To find a point $A \in L$, try $x=0$:

$$(0,y,z) \in P \cap Q \iff \begin{cases} -3y + z = 6 & (\mathrm{I}) \\ -10 + y - z = 4 & (\mathrm{II}) \end{cases}$$

Solve (I) for z: $z = 6 + 3y$. (III)

Substitute (III) into (II): $-10 + y - (6+3y) = 4$.

Solve for y: $y = -10$ (IV)

Substitute (IV) into (III): $z = -24$

So, $(0,-10,-24) \in L \parallel \langle 2,6,16 \rangle$.

Finally, we can parametrize L:

$$x = 0 + 2t, \quad y = -10 + 6t, \quad z = -24 + 16t.$$

(There are many correct parametrizations, not just that one.) ~~but they all have the same simplified symmetric equations.~~

(3) Try $x=y=0$: $\quad 2z = 1$
Try $x=z=0$: $\quad -4y = 1$
Try $y=z=0$: $\quad 5x = 1$

So, 3 points satisfying $5x-4y+2z=1$ are $(1/5,0,0)$, $(0,-1/4,0)$, $(0,0,1/2)$.

(4) $(1,1,1), (3,3,3) \in$ plane $P \parallel \langle -1,-4,-7 \rangle$

So, $P \parallel \langle 3,3,3 \rangle - \langle 1,1,1 \rangle = \langle 2,2,2 \rangle$.
So, $P \perp \langle -1,-4,-7 \rangle \times \langle 2,2,2 \rangle = \langle 6,-12,6 \rangle$

$$P = \{ (x,y,z) \mid \underbrace{\langle x,y,z \rangle - \langle 1,1,1 \rangle \perp \langle 6,-12,6 \rangle} \}$$

$$\iff 6(x-1) - 12(y-1) + 6(z-1) = 0$$

$$\iff \boxed{x + z = 2y}$$

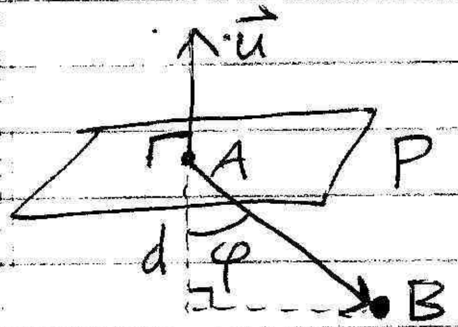
HW10. point $B = (2,3,4)$.

plane $P = \{(x,y,z) \mid 1x - 1y + 1z = 7\}$. ~~plane~~

line L contains $(1,0,0)$ & $(5,0,5)$.

1. At $x=y=0$, $(x,y,z) \in P \Longleftrightarrow z = 7$.

So, $A=(0,0,7) \in P \perp \langle 1,-1,1\rangle = \vec{u}$

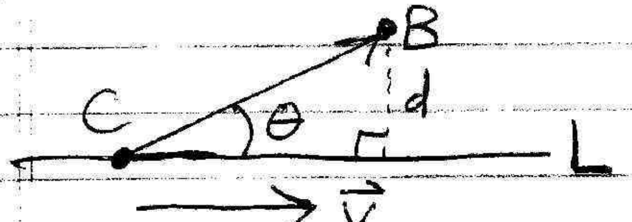


$$d(P,B) = |\overrightarrow{AB}| \cos\varphi = |\overrightarrow{AB} \cdot \vec{u}| \,/\, |\vec{u}|$$

$$= \frac{|\langle 2,3,-3\rangle \cdot \langle 1,-1,1\rangle|}{\sqrt{1^2+1^2+1^2}}$$

$$= |-4|/\sqrt{3} = 4/\sqrt{3}$$

2. $C=(1,0,0) \in L \parallel \vec{v} = \langle 5,0,5\rangle - \langle 1,0,0\rangle$



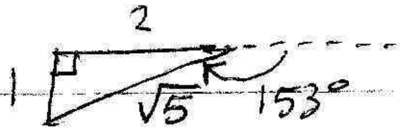
$$d(B,L) = |\overrightarrow{CB}| \sin\theta = \frac{|\overrightarrow{CB} \times \vec{v}|}{|\vec{v}|}$$

$$d(B,L) = |\langle 1,3,4\rangle \times \langle 4,0,5\rangle| \,/\, \sqrt{4^2+0^2+5^2}$$

$$= |\langle 15,11,-12\rangle| \,/\, \sqrt{41}$$

$$= \sqrt{490/41} \approx 3.457$$

HW 11



① $(x, y, z) = (-2, -1, 6)$

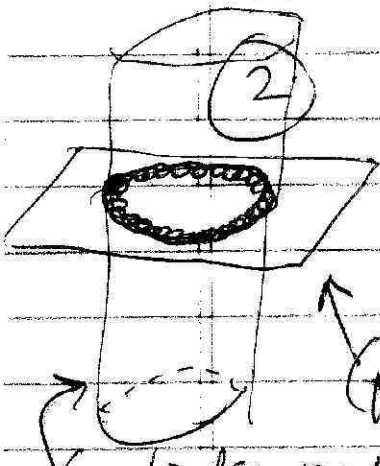
$(r, \theta, z) = (\sqrt{5}, -\cos^{-1}(-2/\sqrt{5}), 6)$

$\approx (2.236, \underbrace{-2.678}_{\text{radians!}}, 6)$

Note: $\alpha = \left(\frac{180\alpha}{\pi}\right)^\circ$ & $\beta^\circ = \frac{\pi\beta}{180}$

In particular, $-2.678 \approx -153^\circ$.

Adding $2\pi (= 360^\circ)$, or any integer multiple $\pm 2\pi, \pm 4\pi, \pm 6\pi, \pm 8\pi, \dots$ to θ also produces a correct answer.



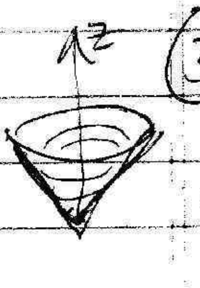
② $r = 4 = z$ is a $\boxed{\text{circle}}$ of radius 4 centered on $(0,0,4)$ and perpendicular to the z-axis.

(plane $z=4$)

(cylinder $r=4$)

In mathematics,
- circle = hollow circle;
- disk = solid circle;
- sphere = hollow sphere;
- ball = solid sphere.

$\varphi = \cos^{-1}(z/\sqrt{z^2+r^2}) = \cos^{-1}(r/\sqrt{2r^2}) = \cos^{-1}(1/\sqrt{2})$



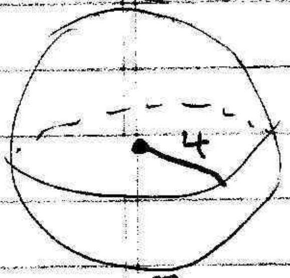
③ $z = r$ is an upwarding opening $\boxed{\text{cone}}$ with vertex $(0,0,0)$ that makes a $\varphi = \underbrace{45^\circ}_{\cos^{-1}(1/\sqrt{2}) = \pi/4}$ angle with the $+z$-axis.

HW12

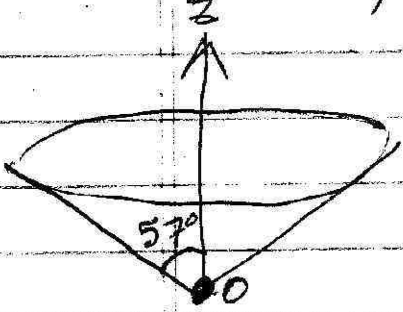
(1) $(x,y,z) = (-5,-4,-3)$
$(r,\theta,z) \approx (6.403, -2.467, -3)$
$(\rho,\varphi,\theta) \approx (7.071, 2.009, -2.467)$

(2) $(\rho,\varphi,\theta) = (8, 2, -1)$
$(r,\theta,z) \approx (7.274, -1, -3.329)$
$(x,y,z) \approx (3.930, -6.121, -3.329)$

(3) a) $\rho = 4$ is a $\boxed{\text{sphere}}$ of radius 4 centered about the origin.



b) $\varphi = 1$ is an upward-opening $\boxed{\text{cone}}$ with vertex at the origin & making a $\underbrace{\approx 57^\circ}_{1 \text{ rad}}$ angle with its axis, the +z-axis.



c) $\varphi = 3$ is a downward-opening $\boxed{\text{cone}}$ with vertex $(0,0,0)$ and $\underbrace{\approx 8^\circ}_{\pi - 3}$ angle with the −z-axis.