HW19

(1) bowl (2) bowl (3) saddle

HW20 (1) The limit does not exist.

For example, as $y = x \to 0^+$ (a diagonal approach $\swarrow$), $\frac{x+y}{x^2+y^2} = \frac{2x}{2x^2} = \frac{1}{x} \to \infty$.

(2) The limit is 0. Using polar coordinates, $\frac{x^3+y^3}{x^2+y^2} = \frac{r^3(\cos^3\theta + \sin^3\theta)}{r^2}$

$= r(\cos^3\theta + \sin^3\theta)$ is between $-2r$ & $2r$, which both tend to 0 as $(x,y) \to (0,0)$.

(3) The limit does not exist.

Along the line $y = x$, as $y = x \to 0$,

$\frac{x^3+y^2}{x^2+y^2} = \frac{x^2(x+1)}{2x^2} = \frac{1}{2}(x+1) \to \frac{1}{2}$; along the line $y=0$, as $(x,y) = (x,0) \to (0,0)$,

$\frac{x^3+y^2}{x^2+y^2} = \frac{x^3}{x^2} = x \to 0 \neq \frac{1}{2}$.

(4) $\dfrac{x^2+x^2y}{x^4+y^2+2y-1} \to \dfrac{0^2+0^2(-1)}{0^4+(-1)^2+2(-1)-1}$

$= \dfrac{0}{-2} = 0.$ (No division by 0? No problem!)

HW21 $|(2x-4y+4)-(2\cdot 6-4\cdot 3+4)|$

$= |2(x-6)-4(y-3)|$

$\leq 2|x-6|+4|y-3|$

$< 2(\varepsilon/6)+4(\varepsilon/6)$

$= \varepsilon$ <u>if</u> $|x-6|, |y-3| < \varepsilon/6$

The above works for any $\varepsilon > 0$.

So, $\lim\limits_{(x,y)\to(6,3)} (2x-4y+4) = 2\cdot 6-4\cdot 3+4.$

Note: you really need $\varepsilon/6$ or smaller because, e.g., if $(x,y)=(2.01, 3.99)$, then $(2x-4y+4)-(2\cdot 6-4\cdot 3+4) = 0.06.$

HW22

1. $r_x = \dfrac{x}{\sqrt{x^2+y^2}}$; $r_y = \dfrac{y}{\sqrt{x^2+y^2}}$; $\left(r=\sqrt{x^2+y^2}\right)$

$\left(z=\dfrac{\sin r}{r}\right)$

$$z_x = \frac{\left(\sin\sqrt{x^2+y^2}\right)_x \sqrt{x^2+y^2} - \left(\sin\sqrt{x^2+y^2}\right)\left(\sqrt{x^2+y^2}\right)_x}{\left(\sqrt{x^2+y^2}\right)^2}$$

$$= \left[\left(\cos\sqrt{x^2+y^2}\right)(x) - \left(\sin\sqrt{x^2+y^2}\right) x/\sqrt{x^2+y^2}\right]/\left(x^2+y^2\right)$$

$$= [r\cos r - \sin r] \times r^{-3}$$

$$z_y = \frac{\left(\sin\sqrt{x^2+y^2}\right)_y \sqrt{x^2+y^2} - \left(\sin\sqrt{x^2+y^2}\right)\left(\sqrt{x^2+y^2}\right)_y}{\left(\sqrt{x^2+y^2}\right)_y}$$

$$= \cdots = [r\cos r - \sin r]\, y r^{-3}.$$

2. $x=3$ & $z - \dfrac{\sin\sqrt{3^2+4^2}}{\sqrt{3^2+4^2}} = z_y(3,4)(y-4)$.

At $(x,y)=(3,4)$, $r=5$. So, the line is

$$\left\{(x,y,z) \;\middle|\; x=3 \;\&\; z-\frac{\sin 5}{5} = [5\cos 5 - \sin 5]\left(\frac{4}{125}\right)(y-4)\right\}.$$

HW23) $z=f(x,y)=5-x^2-xy-y^2 = 2$ @ $(1,1)$

$f_x = 0-2x-y-0 = -3$ @ $(1,1)$

$f_y = 0-0-x-2y = -3$ @ $(1,1)$

Tangent plane : $z = 2+(-3)(x-1)+(-3)(y-1)$

At $(1.002, 1.003)$, $z=f=1.984981$.

At $(1.002, 1.003)$, the tangent plane z

is 1.985. (Pretty close!)