

HW19

- ① bowl    ② bowl    ③ saddle

HW20 ① The limit does not exist.

For example, as  $y = x \rightarrow 0^+$  (a diagonal

approach:  $\swarrow$ ),  $\frac{x+y}{x^2+y^2} = \frac{2x}{2x^2} = \frac{1}{x} \rightarrow \infty$ .

② The limit is 0. Using polar coordinates,  $\frac{x^3+y^3}{x^2+y^2} = \frac{r^3(\cos^3\theta + \sin^3\theta)}{r^2}$

$= r(\cos^3\theta + \sin^3\theta)$  is between  $-2r$  &  $2r$ , which both tend to 0 as  $(x,y) \rightarrow (0,0)$ .

③ The limit does not exist.

Along the line  $y=x$ , as  $y=x \rightarrow 0$ ,

$$\frac{x^3+y^2}{x^2+y^2} = \frac{x^2(x+1)}{2x^2} = \frac{1}{2}(x+1) \rightarrow \frac{1}{2}; \text{ along}$$

the line  $y=0$ , as  $(x,y) = (x,0) \rightarrow (0,0)$ ,

$$\frac{x^3+y^2}{x^2+y^2} = \frac{x^3}{x^2} = x \rightarrow 0 \neq \frac{1}{2}.$$

$$\textcircled{4} \quad \frac{x^2 + x^2 y}{x^4 + y^2 + 2y - 1} \rightarrow \frac{0^2 + 0^2(-1)}{0^4 + (-1)^2 + 2(-1) - 1}$$

$$= \frac{0}{-2} = 0. \quad (\text{No division by } 0?)$$

No problem!

$$\text{HW 21} \quad |(2x - 4y + 4) - (2 \cdot 6 - 4 \cdot 3 + 4)|$$

$$= |2(x - 6) - 4(y - 3)|$$

$$\leq 2|x - 6| + 4|y - 3|$$

$$< 2(\varepsilon/6) + 4(\varepsilon/6)$$

$$= \varepsilon \quad \text{if} \quad |x - 6|, |y - 3| < \textcircled{\varepsilon/6}$$

The above works for any  $\varepsilon > 0$ .

$$\text{So,} \quad \lim_{(x,y) \rightarrow (6,3)} (2x - 4y + 4) = 2 \cdot 6 - 4 \cdot 3 + 4.$$

Note: you really need  $\textcircled{\varepsilon/6}$  or smaller

because, e.g., if  $(x,y) = (2.01, 3.99)$ , then

$$(2x - 4y + 4) - (2 \cdot 6 - 4 \cdot 3 + 4) = 0.06.$$

HW22

①

$$r_x = \frac{x}{\sqrt{x^2+y^2}} ; \quad r_y = \frac{y}{\sqrt{x^2+y^2}} ; \quad (r = \sqrt{x^2+y^2})$$

$$z_x = \frac{(\sin \sqrt{x^2+y^2}) x \sqrt{x^2+y^2} - (\sin \sqrt{x^2+y^2})(\sqrt{x^2+y^2}) x}{(\sqrt{x^2+y^2})^2} \quad (z = \frac{\sin r}{r})$$

$$= \frac{(\cos \sqrt{x^2+y^2}) (x) - (\sin \sqrt{x^2+y^2}) x / \sqrt{x^2+y^2}}{(x^2+y^2)}$$

$$= [r \cos r - \sin r] x r^{-3}$$

$$z_y = \frac{(\sin \sqrt{x^2+y^2}) y \sqrt{x^2+y^2} - (\sin \sqrt{x^2+y^2})(\sqrt{x^2+y^2}) y}{(\sqrt{x^2+y^2})^2}$$

$$= \dots = [r \cos r - \sin r] y r^{-3}$$

②  $x=3$  &  $z = \frac{\sin \sqrt{3^2+4^2}}{\sqrt{3^2+4^2}} = z_y(3,4)(y-4)$ ,

At  $(x,y) = (3,4)$ ,  $r=5$ . So, the line is

$$\left\{ (x,y,z) \mid x=3 \text{ \& } z = \frac{\sin 5}{5} = [5 \cos 5 - \sin 5] \left( \frac{y}{125} \right) (y-4) \right\}$$

HW2.3 }  $z = f(x, y) = 5 - x^2 - xy - y^2 = 2$  @ (1, 1)

$$f_x = 0 - 2x - y - 0 = -3 \quad @ (1, 1)$$

$$f_y = 0 - 0 - x - 2y = -3 \quad @ (1, 1)$$

Tangent plane :  $z = 2 + (-3)(x-1) + (-3)(y-1)$

At (1.002, 1.003),  $z = f = 1.984981$ .

At (1.002, 1.003), the tangent plane  $z$

is 1.985. (Pretty close!)