HW30 Objective: $\overbrace{(x-1)^2+(y-2)^2+(z-1)^2}^{f(x,y,z)=\text{ squared distance}}$

Constraint: $g(x,y,z) = \left(\frac{x}{7}\right)^2+\left(\frac{y}{3}\right)^2+z^2-1=0$

$$\vec{\nabla} f = \langle 2(x-1),\ 2(y-2),\ 2(z-1)\rangle$$

$$\vec{\nabla} g = \langle 2x/49,\ 2y/9,\ 2z\rangle$$

Solve $\begin{cases} \vec{\nabla} f = \lambda \vec{\nabla} g \\ g = 0 \end{cases}$: $\begin{cases} 49(x-1)=\lambda x \\ 9(y-2)=\lambda y \\ z-1=\lambda z \\ (x/7)^2+(y/3)^2+z^2=1 \end{cases}$

$$\Rightarrow \begin{cases} x = 49/(49-\lambda) \\ y = 18/(9-\lambda) \\ z = 1/(1-\lambda) \\ 1 = x^2/49+y^2/9+z^2 \end{cases}$$

$$\Rightarrow 1 = \frac{49}{(\lambda-49)^2}+\frac{36}{(\lambda-9)^2}+\frac{1}{(\lambda-1)^2}$$

My calculator's solve command found me four solutions:

$\lambda \approx -.329,\ 15.149,\ 41.878,\ 56.059.$

The corresponding (x,y,z) solutions are:

① $=(.99333,\ 1.9295,\ .75247)$
② $=(1.4475,\ -2.9275,\ -.070678)$
③ $=(6.88028,\ -.54748,\ -.024463)$
④ $=(-6.9417,\ -.38250,\ -.018162)$

The distances $\sqrt{f(x,y,z)}$ from these four "constrained critical points" to $(1,2,1)$ are:

point	distance	
1	0.25747	$\Rightarrow A \approx (.99, 1.93, .75)$
2	5.0623	
3	6.4898	
4	8.3536	$\Rightarrow B \approx (-6.94, -.38, -.02)$

So, $A = 1$ is the closest point on the ellipsoid, with distance $\approx .25$; $B = 4$ is the farthest, ≈ 8.35 away.

HW32 (1) $\displaystyle\iint_{[1,2]\times[1,3]} \frac{x^2}{y}\,dx\,dy$

$$= \int_1^2 x^2\,dx \int_1^3 \frac{dy}{y} = \frac{x^3}{3}\Big|_1^2 \cdot \ln\frac{3}{1}$$

$$= \boxed{(7/3)\ln 3}$$

(2) $$\frac{1}{(\pi-0)(\pi-0)} \int_0^\pi \left[\int_0^\pi \sin^2(x+y)\,dy\right] dx$$

$$= \pi^{-2}\int_0^\pi \left[\left(\frac{y}{2} - \frac{\sin(2(x+y))}{4}\right)\Big|_{y=0}^{y=\pi}\right] dx$$

$$= \pi^{-2}\int_0^\pi \left(\frac{\pi}{2}\right) dx = \boxed{\frac{1}{2\pi}}$$

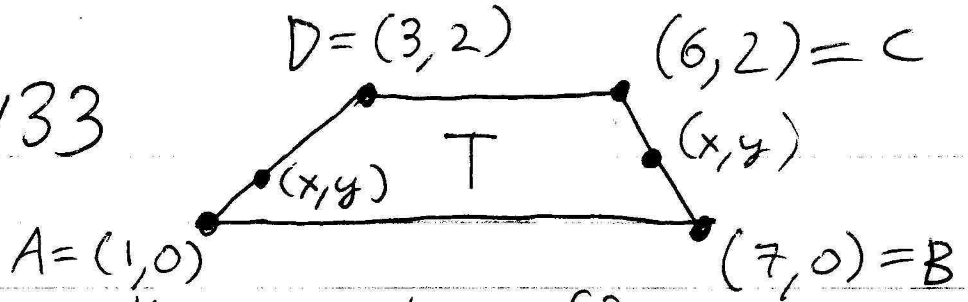
(Yes, you ~~cam~~ just do all these integrals in the calkulator.)

(3) $$\iint_{[0,4]\times[-2,5]} e^{-x+y/2}\,dx\,dy$$

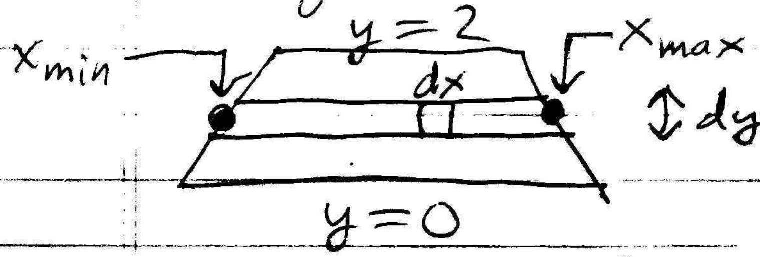
$$= \int_0^4 e^{-x}\,dx \int_{-2}^5 e^{y/2}\,dy$$

$$= \boxed{(e^0 - e^{-4})(e^{5/2} - e^{-1})(2)}$$

HW33



We will compute $\iint_T \cdots$ integrals using horizontal slicing:



$$\iint_T = \int_{y=0}^{y=2} \int_{x=x_{min}}^{x=x_{max}}$$

line AD: $\dfrac{y-0}{x-1} = \dfrac{2-0}{3-1} \Rightarrow x_{min} = 1+y$

line BC: $\dfrac{y-0}{x-7} = \dfrac{2-0}{6-7} \Rightarrow x_{max} = 7 - \dfrac{y}{2}$

$$\iint_T \ldots dx\,dy = \int_0^2 \left(\int_{1+y}^{7-y/2} \ldots dx \right) dy$$

$$\iint_T 1\,dx\,dy = 9$$

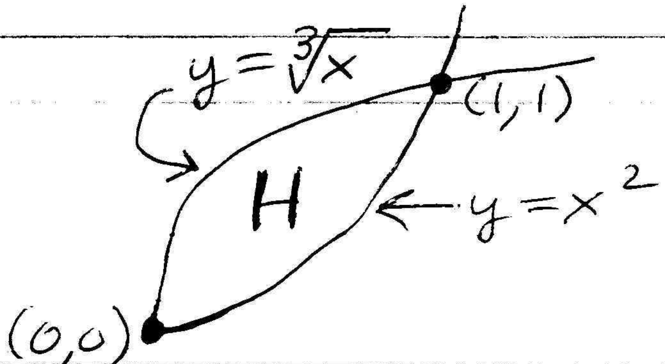
$$\iint_T x\,dx\,dy = 38$$

$$\iint_T y\,dx\,dy = 8$$

$$(x_{cm}, y_{cm}) = \left(\frac{38}{9}, \frac{8}{9} \right)$$

HW34 $\dfrac{\iint_H (x^2+y^2)\,dx\,dy}{\iint_H 1\,dx\,dy}$ = average value of r^2 on region H,

...where H is bounded by $y=x^2$ and $x=y^3$:



vertical slicing

$$\iint_H f(x,y)\,dx\,dy = \int_0^1 \left(\int_{x^2}^{\sqrt[3]{x}} f(x,y)\,dy \right) dx$$

$$\frac{\iint_H x^2+y^2\,dx\,dy}{\iint_H 1\,dx\,dy} = \frac{23/105}{5/12} = \boxed{\frac{92}{175}}$$

If you use horizontal slicing, then

$$\iint_H = \int_{y=0}^{y=1} \int_{x=y^3}^{x=\sqrt{y}} .$$

Your final answer will be the same.