HW35

$$\text{area}(M) = \int_0^{2\pi} \overbrace{\left(\int_\theta^{\theta+\theta^2} r \, dr \right)}^{dA} d\theta$$

$$= 4\pi^4 + \frac{16\pi^5}{5} \approx 1369$$

$$\iint_M x \, dA = \int_0^{2\pi} \left[\int_\theta^{\theta+\theta^2} r(\cos\theta) \, r \, dr \right] d\theta$$

$$= -288\pi^3 - 240\pi^2 + 432\pi + 80\pi^4 + 64\pi^5$$

$$\approx 17437$$

$$\iint_M y \, dA = \int_0^{2\pi} \left[\int_\theta^{\theta+\theta^2} r(\sin\theta) \, r \, dr \right] d\theta$$

$$= -\frac{64}{3}\pi^6 - 32\pi^5 - 432\pi^2 - 240\pi$$

$$+ 160\pi^3 + 144\pi^4 \approx -16332$$

$$\iint_M \sqrt{x^2+y^2} \, dA = \int_0^{2\pi} \left[\int_\theta^{\theta+\theta^2} r \, r \, dr \right] d\theta$$

$$= \frac{32}{5}\pi^5 + \frac{32}{3}\pi^6 + \frac{128}{21}\pi^7$$

$$\approx 30623$$

$$\left[\text{average } x \text{ in } M\right] = \iint_M x\, dA \Big/ \text{area}(M)$$
$$\approx \boxed{12.74}$$

$$\left[\text{avg. } y \text{ in } M\right] = \iint_M y\, dA \Big/ \text{area}(M)$$
$$\approx \boxed{-11.93}$$

$$\left[\text{avg. } \sqrt{x^2+y^2} \text{ in } M\right] = \iint_M r\, dA \Big/ \text{area}(M)$$
$$\approx \boxed{22.37}$$

HW36

① $$\iiint_{[0,2]\times[0,3]\times[1,4]} \overbrace{\underbrace{\sqrt{x^5y^3z}}_{dm/dV}\ \underbrace{dx\,dy\,dz}_{dV}}^{dm}$$ ← This integral is the total mass.

$$= \int_0^2 x^{5/2}\,dx \int_0^3 y^{3/2}\,dy \int_1^4 z^{1/2}\,dz$$

$$= \frac{2^{7/2}}{7/2} \cdot \frac{3^{5/2}}{5/2} \cdot \frac{4^{3/2}-1}{3/2} \approx \boxed{94.06}$$

② $$\iiint_{[0,\pi]^3} \sin^2(x+2y-3z)\,dV = \int_0^\pi\left(\int_0^\pi\left(\int_0^\pi \sin^2(x+2y+3z)\,dx\right)dy\right)dz$$

$$= \boxed{\pi^3/2 \approx 15.50}$$

HW37 Let $N = \{(x,y,z) \mid 0 \le x \le z \le y \le 4\}$.

$$\iiint_N = \int_{y=0}^{y=4} \int_{z=0}^{z=y} \int_{x=0}^{x=z} \quad \text{or} \quad \int_{x=0}^{x=4} \int_{z=x}^{z=4} \int_{y=z}^{y=4}$$

$$\text{or} \quad \int_{z=0}^{z=4} \int_{x=0}^{x=z} \int_{y=z}^{y=4} \quad \text{or} \quad \int_{z=0}^{z=4} \int_{y=z}^{y=4} \int_{x=0}^{x=z}$$

$$\text{or} \quad \int_{y=0}^{y=4} \int_{x=0}^{x=y} \int_{z=x}^{z=y} \quad \text{or} \quad \int_{x=0}^{x=4} \int_{y=x}^{y=4} \int_{z=x}^{z=y}.$$

Take your pick. I like the first one best.

$$\iiint_N dm = \int_0^4 \left[\int_0^y \left[\int_0^z xy \, dx \right] dz \right] dy = \frac{512}{15}$$

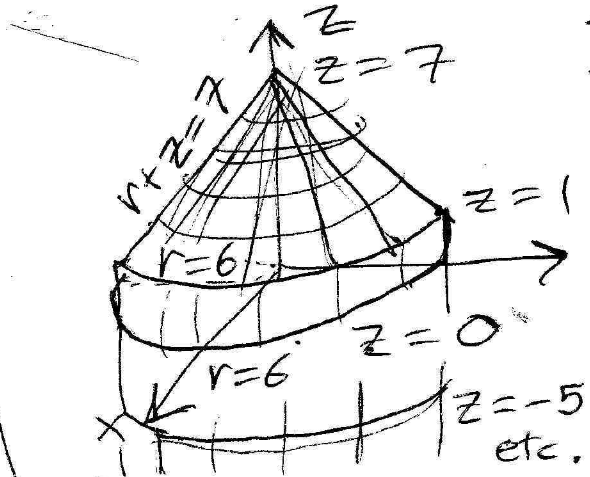
(The other five slicing strategies produce $\frac{512}{15}$ too. I checked.)

$$\iiint_N (x^2+z^2)\, dm = \int_0^4 \left(\int_0^y \left(\int_0^z xy(x^2+z^2)\, dx \right) dz \right) dy$$

$= 12288/35$

$$\begin{bmatrix} \text{average of } x^2+z^2 \\ \text{in } N \end{bmatrix} = \frac{12288/35}{512/15} = \boxed{\frac{72}{7}} \approx 10.286$$

$K = \{(x,y,z) \mid 0 \le y \ \& \ r \le 6 \ \& \ \underbrace{0 \le \theta \le \pi}_{\text{implies } 0 \le y} \ \& \ \underbrace{r+z \le 7}_{\text{boundary } r+z=7}\}$



Intended half-cone has $z \ge 1$.

HW38

Sorry, this region has infinite volume because z can go arbitrarily low. For example, $(r,\theta,z) = (5, \pi/2, -1000) \in K$. I should have specified another boundary like $1 \le z$. If I had required $1 \le z$, then

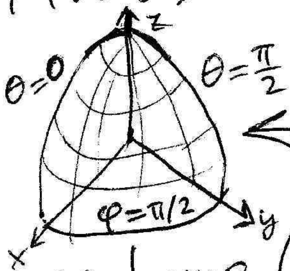
$$\begin{bmatrix}\text{average } \theta \\ \text{in } K\end{bmatrix} = \int_0^{\pi}\int_0^{6}\int_1^{7} \theta \underbrace{dz\, r\, dr\, d\theta}_{dV} \Big/ \int_0^{\pi}\int_0^{6}\int_1^{7} \underbrace{dz\, r\, dr\, d\theta}_{dV}$$

$$\& \begin{bmatrix}\text{average } \theta^2 \\ \text{in } K\end{bmatrix} = \int_0^{\pi}\int_0^{6}\int_1^{7} \theta^2 \underbrace{dz\, r\, dr\, d\theta}_{dV} \Big/ \int_0^{\pi}\int_0^{6}\int_1^{7} \underbrace{dz\, r\, dr\, d\theta}_{dV}$$

$$\left[\text{avg. } \theta\right] = \frac{\int_0^{\pi} \theta\, d\theta \cancel{\int_0^6 r\,dr} \cancel{\int_1^7 dz}}{\int_0^{\pi} d\theta \cancel{\int_0^6 r\,dr} \cancel{\int_1^7 dz}} = \frac{\pi^2/2}{\pi} = \frac{\pi}{2}$$

$$\left[\text{avg. } \theta^2\right] = \frac{\int_0^{\pi} \theta^2\, d\theta \cancel{\int_0^6 r\,dr} \cancel{\int_1^7 dz}}{\int_0^{\pi} d\theta \cancel{\int_0^6 r\,dr} \cancel{\int_1^7 dz}} = \frac{\pi^3/3}{\pi} = \frac{\pi^2}{3}$$

HW39 ① $\iiint_M = \underbrace{\int_{\rho=0}^{\rho=6} \int_{\theta=0}^{\theta=\pi/2} \int_{\varphi=0}^{\varphi=\pi/2}}_{\text{A spherical "box"}}$



$\leftarrow$ (octant M)

volume$(M) = \iiint_M 1\, dV$ where

$dV = \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$. But we can also use geometry: volume $= \frac{1}{8} \cdot \frac{4}{3}\pi \cdot 6^3$.

Either way, volume $= 36\pi$.

$$\iiint_M x\, dV = \iiint_M (\rho \sin\varphi \cos\theta)(\rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta)$$

$$= \int_0^6 \rho^3\, d\rho \int_0^{\pi/2} \sin^2\varphi\, d\varphi \int_0^{\pi/2} \cos\theta\, d\theta$$

$$= (6^4/4)(\pi/4)(1) = 81\pi$$

$$[\text{avg. } x \text{ in } M] = 81/36 = \boxed{9/4}$$

② $\iiint_M \rho^{-2}\, dV = \int_0^6 d\rho \int_0^{\pi/2} \sin\varphi\, d\varphi \int_0^{\pi/2} d\theta = \boxed{3\pi}$