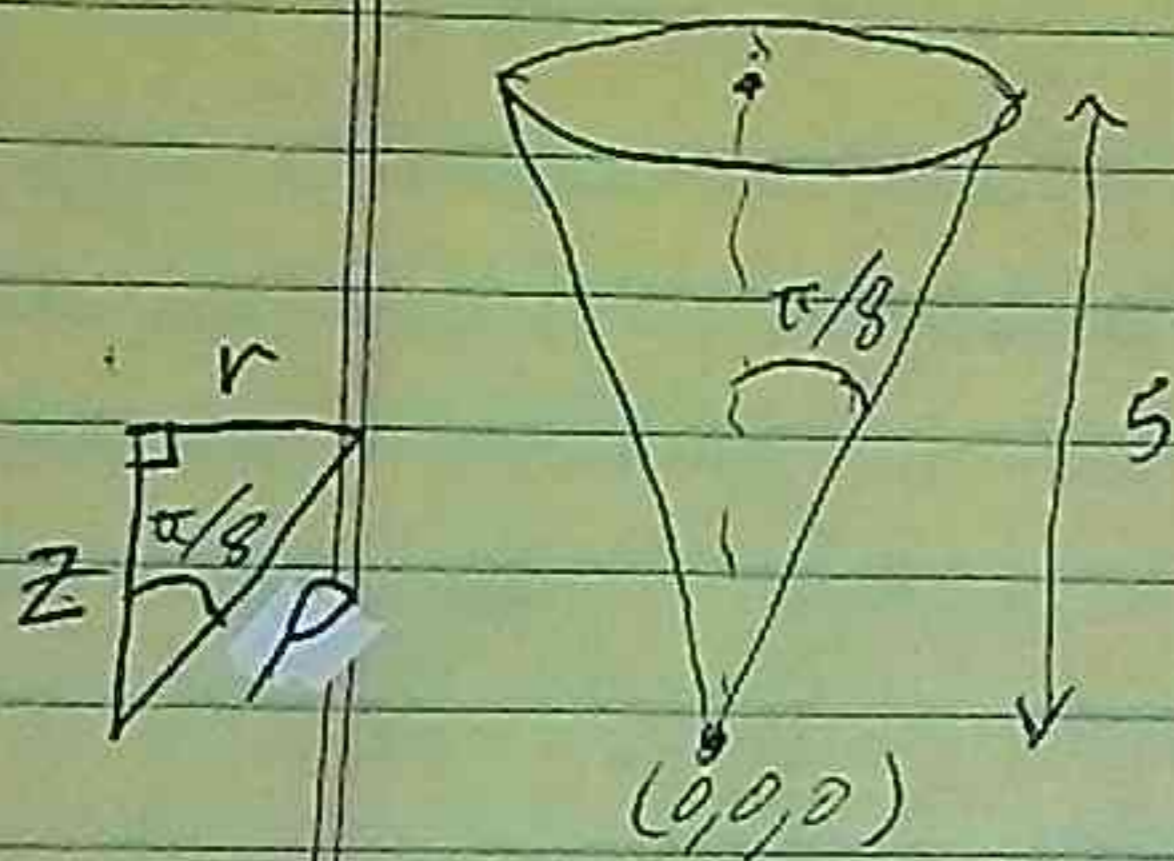


(Assume density  $dm/dV = 1$ .)

HW4/ Let  $C = \{(x, y, z) \mid \varphi \leq \frac{\pi}{8} \text{ \& } z \leq 5\}$ .



Boundary surfaces:

$$\varphi = \pi/8$$

$$\Leftrightarrow r = z \tan(\pi/8)$$

$$\& \rho \cos \varphi = z = 5$$

$$\Leftrightarrow \rho = 5 / \cos \varphi (= 5 \sec \varphi)$$

$$\iiint_C = \int_{\theta=0}^{\theta=2\pi} \int_{\varphi=0}^{\varphi=\pi/8} \int_{\rho=0}^{\rho=5/\cos \varphi}$$

$$\text{or } \int_{\theta=0}^{\theta=2\pi} \int_{z=0}^{z=5} \int_{r=0}^{r=z \tan(\pi/8)}$$

$$\text{or } \int_{\theta=0}^{\theta=2\pi} \int_{z=0}^{z=5} \int_{r=0}^{r=z \tan(\pi/8)} r^2 dr dz d\theta$$

$$I_x = \iiint_C (y^2 + z^2) dV$$

$$I_y = \iiint_C (x^2 + z^2) dV$$

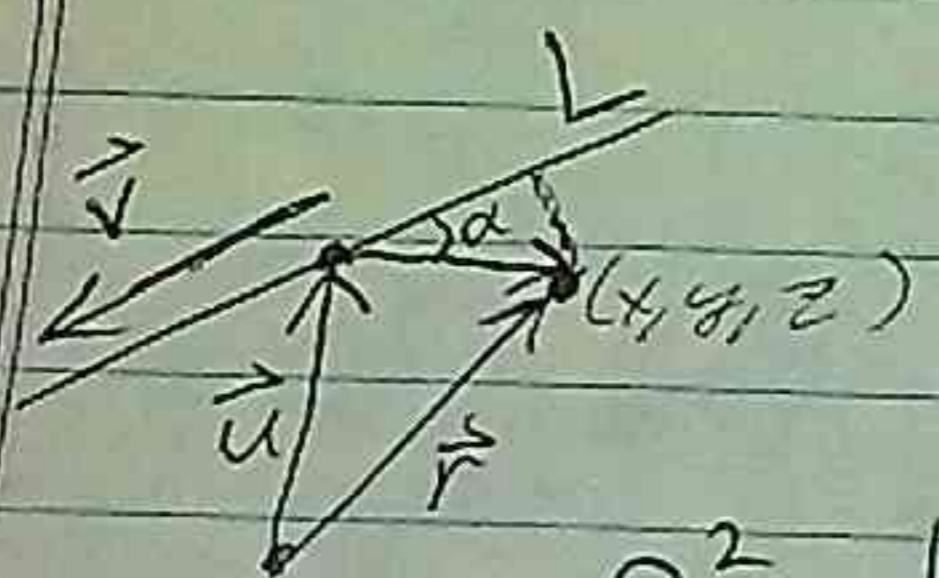
$$I_z = \iiint_C (x^2 + y^2) dV$$

$$I_L = \iiint_C R^2 dV$$

where  $R$  is distance from  $(x, y, z)$  to line  $L$ .

← slightly easier, but it won't matter to the calculator.

$$L = \{ (x, y, z) \mid \underbrace{\langle x, y, z \rangle}_{\vec{r}} = \underbrace{\langle 1, 2, -3 \rangle}_{\vec{u}} + t \underbrace{\langle 1, -1, 1 \rangle}_{\vec{v}} \}$$



$$R = |\vec{r} - \vec{u}| \sin \alpha = \frac{|(\vec{r} - \vec{u}) \times \vec{v}|}{|\vec{v}|}$$

$$R^2 = \frac{|(\vec{r} - \vec{u}) \times \vec{v}|^2}{|\vec{v}|^2}$$

$$R^2 = \frac{1}{3} \left( (y + z + 1)^2 + (z - x + 4)^2 + (3 - x - y)^2 \right)$$

$$I_x = \int_0^{2\pi} \int_0^5 \int_0^{\tan \pi/8} \overbrace{(r^2 \sin^2 \theta + z^2)}^{y^2 + z^2} \overbrace{r \, dr \, dz \, d\theta}^{dV}$$

$$I_y = \int_0^{2\pi} \int_0^5 \int_0^{\tan \pi/8} \overbrace{(r^2 \cos^2 \theta + z^2)}^{x^2 + z^2} \overbrace{r \, dr \, dz \, d\theta}^{dV}$$

$$I_z = \int_0^{2\pi} d\theta \cdot \int_0^5 \int_0^{\tan \pi/8} \overbrace{\frac{x^2 + y^2}{r^2}}^{1} \cdot \overbrace{r \, dr \, dz}^{dV}$$

$$I_L = \int_0^{2\pi} \int_0^5 \int_0^{\tan \pi/8} \frac{1}{3} \left[ (r \sin \theta + z + 1)^2 + (z - r \cos \theta + 4)^2 + (3 - r \cos \theta - r \sin \theta)^2 \right] r \, dr \, dz \, d\theta$$

$$I_x = I_y = \frac{625\pi}{4} (29 - 20\sqrt{2}) \approx \boxed{351.33}$$

$$I_z = \frac{625\pi}{2} (17 - 12\sqrt{2}) \approx \boxed{28.900}$$

$$I_L = \frac{125\pi}{18} (1071 - 734\sqrt{2}) \approx \boxed{719.23}$$

HW42  $(f, g)$  as given maps

$P$  to  $ABD$  &  $Q$  to  $ABCD$

where  $\begin{cases} Q = \{(u, v) \mid 0 \leq u, v \leq 1\} \\ P = \{(u, v) \mid 0 \leq u, v \leq 1\} \end{cases}$

$$(x, y) = (f, g)(u, v) = (4 + 10u - v, 7 - u + 8v)$$

$$|x_u y_v - x_v y_u| = |10 \cdot 8 - (-1)(-1)| = 79$$

was also given,

$$\iint_{ABD} y^2 dx dy = \iint_P \overbrace{(7 - u + 8v)^2}^{y^2} \overbrace{79}^{dA} du dv$$

$$\iint_{ABD} y^2 dx dy = \iint_P \sqrt{(7-u+8v)^2 + 79} du dv$$

$$= \int_0^1 \left( \int_0^v (7-u+8v)^2 \sqrt{79} du \right) dv$$

$$= \frac{69757}{12} \approx 5813$$

$x^2 \quad dA$

$$\iint_{ABCD} x^2 dx dy = \iint_Q \sqrt{(4+10u-v)^2 + 79} du dv$$

$$= \int_0^1 \left( \int_0^1 (4+10u-v)^2 \sqrt{79} du \right) dv$$

$$= \frac{27413}{3} \approx 9138$$