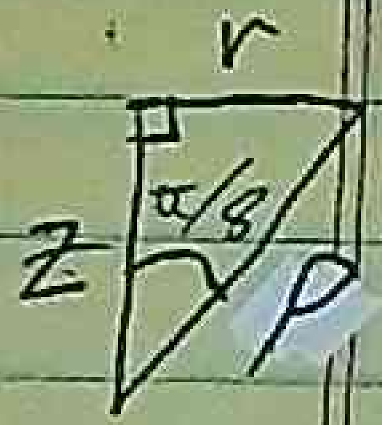
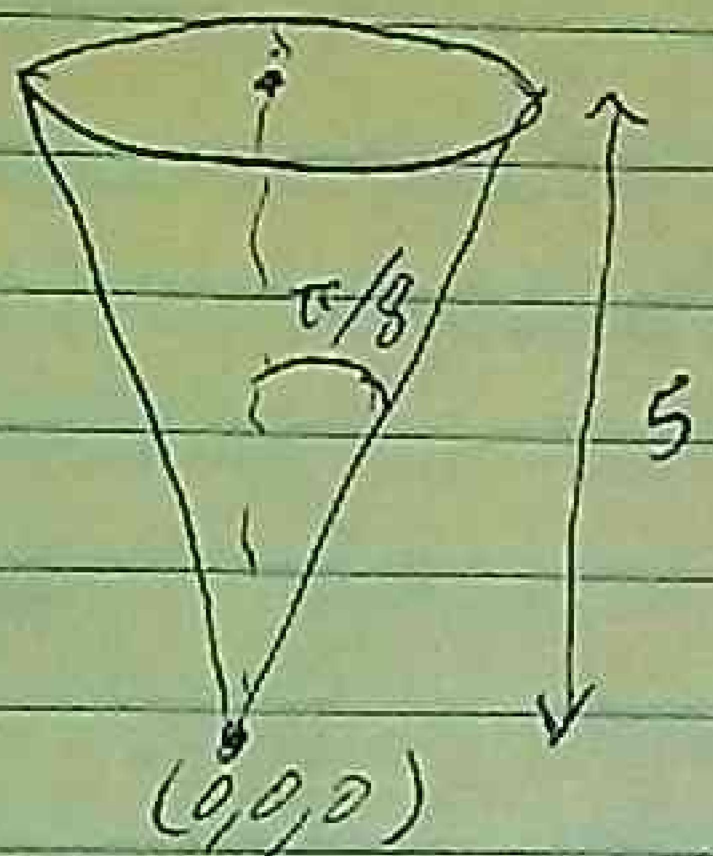
(Assume density $dm/dV = 1$.)

HW41 Let $C = \{(x,y,z) \mid \varphi \le \frac{\pi}{8} \ \& \ z \le 5\}$.





Boundary surfaces:

$$\varphi = \pi/8 \iff r = z\tan(\pi/8)$$

$$\&\ \rho\cos\varphi = z = 5 \iff \rho = 5/\cos\varphi \ (= 5\sec\varphi)$$

$$\iiint_C = \int_{\theta=0}^{\theta=2\pi} \int_{\varphi=0}^{\varphi=\pi/8} \int_{\rho=0}^{\rho=5/\cos\varphi}$$

$$\text{or } \boxed{\int_{\theta=0}^{\theta=2\pi} \int_{z=0}^{z=5} \int_{r=0}^{r=z\tan(\pi/8)}}$$

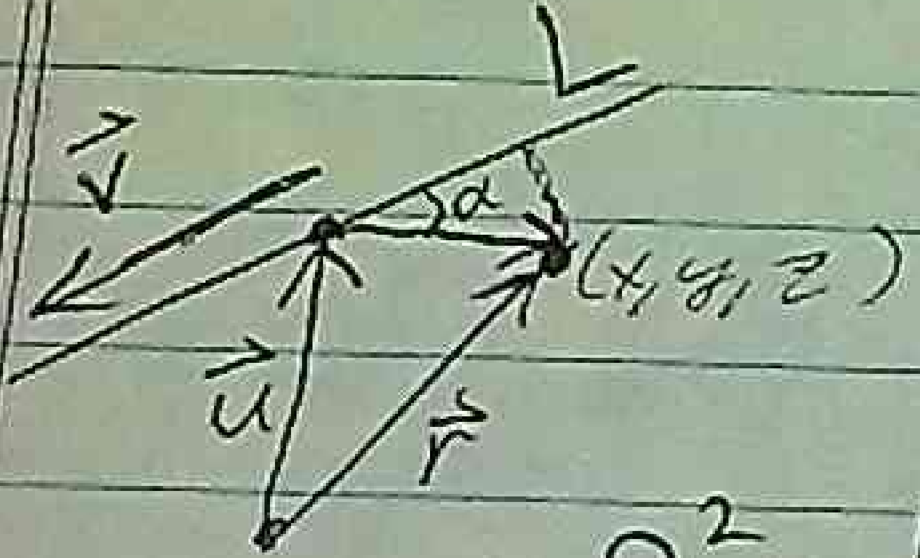
or $\boxed{\int_{\theta=0}^{\theta=2\pi}\int_{z=0}^{z=5}\int_{r=0}^{r=z\tan(\pi/8)}}$

slightly easier, but it won't matter to the calculator.

$$I_x = \iiint_C (y^2+z^2)\,dV$$
$$I_y = \iiint_C (x^2+z^2)\,dV$$
$$I_z = \iiint_C (x^2+y^2)\,dV$$
$$I_L = \iiint_C R^2\,dV$$

where R is distance from (x,y,z) to line L.

$$L = \{(x,y,z) \mid \underbrace{\langle x,y,z\rangle}_{\vec r} = \underbrace{\langle 1,2,-3\rangle}_{\vec u} + t\underbrace{\langle 1,-1,1\rangle}_{\vec v}\}$$



$$R = |\vec r-\vec u|\sin\alpha = |(\vec r-\vec u)\times\vec v|/|\vec v|$$
$$R^2 = |(\vec r-\vec u)\times\vec v|^2/|\vec v|^2$$
$$R^2 = \frac{1}{3}\left((y+z+1)^2+(z-x+4)^2+(3-x-y)^2\right)$$

$$I_x = \int_0^{2\pi}\left(\int_0^5\left(\int_0^{z\tan\pi/8}\overbrace{(r^2\sin^2\theta + z^2)}^{y^2+z^2}\overbrace{r\,dr}^{dV}\right)dz\right)d\theta$$

$$I_y = \int_0^{2\pi}\left(\int_0^5\left(\int_0^{z\tan\pi/8}\overbrace{(r^2\cos^2\theta + z^2)}^{x^2+z^2}\overbrace{r\,dr}^{dV}\right)dz\right)d\theta$$

$$I_z = \int_0^{2\pi} d\theta \cdot \int_0^5\left(\int_0^{z\tan\pi/8}\overbrace{r^2}^{x^2+y^2}\cdot r\,dr\right)dz$$

$$I_L = \int_0^{2\pi}\left(\int_0^5\left(\int_0^{z\tan\pi/8}\frac{1}{3}\Big[(r\sin\theta + z + 1)^2 + (z - r\cos\theta + 4)^2 + (3 - r\cos\theta - r\sin\theta)^2\Big]r\,dr\right)dz\right)d\theta$$

$$I_x = I_y = \frac{625\pi}{4}(29 - 20\sqrt{2}) \approx \boxed{351.33}$$

$$I_z = \frac{625\pi}{2}(17 - 12\sqrt{2}) \approx \boxed{28.900}$$

$$I_L = \frac{125\pi}{18}(1071 - 734\sqrt{2}) \approx \boxed{719.23}$$

HW42 (f,g) as given maps

P to ABD & Q to $ABCD$

where $\begin{cases} Q = \{(u,v) \mid 0 \le u,v \ \& \ u,v \le 1\} \\ P = \{(u,v) \mid 0 \le u,v \ \& \ u+v \le 1\} \end{cases}$

$(x,y) = (f,g)(u,v) = (4 + 10u - v, \ 7 - u + 8v)$

$|x_u y_v - x_v y_u| = |10 \cdot 8 - (-1)(-1)| = 79$

was also given.

$$\iint_{ABD} y^2 \, dx \, dy = \iint_P \overbrace{(7-u+8v)^2}^{y^2} \overbrace{79 \, du \, dv}^{dA}$$

$$\iint_{ABD} y^2\,dx\,dy = \iint_P \overbrace{(7-u+8v)^2}\ \overbrace{79\,du\,dv}^{dA}$$

$$= \int_0^1 \left(\int_0^v (7-u+8v)^2\, 79\, du \right) dv$$

$$\boxed{= 69757/12 \approx 5813}$$

$$\iint_{ABCD} x^2\,dx\,dy = \iint_Q \overbrace{(4+10u-v)^2}^{x^2}\ \overbrace{79\,du\,dv}^{dA}$$

$$= \int_0^1 \left(\int_0^1 (4+10u-v)^2\, 79\, du \right) dv$$

$$\boxed{= 27413/3 \approx 9138}$$