Hw9 ① $P \parallel \vec{a} = \langle 1,2,3\rangle$ and $B=(0,1,0)\in P \ni (1,0,0)=C$ are given. So, $P \parallel \vec{a}, \vec{b}$ where $\vec{b}=\overrightarrow{BC}=\langle 1,-1,0\rangle$.

So, $(1,0,0)\in P \perp \vec{a}\times\vec{b}=\langle 3,3,-3\rangle$. So, $D\in P \iff \overrightarrow{CD} \perp \vec{a}\times\vec{b}$

$\Updownarrow$

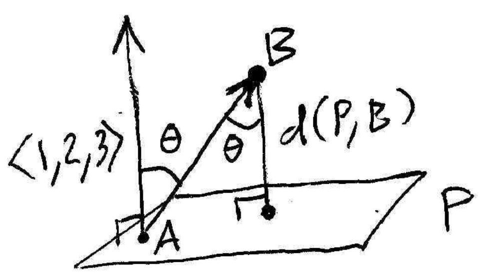
$\overrightarrow{CD}\cdot(\vec{a}\times\vec{b})=0$.

So, $P=\{(x,y,z) \mid \boxed{3(x-1)+3y-3z=0}\}$

(We can optionally simplify to $x+y=z+1$.)

② For $P$ given by $x+2y+3z=5$, we have $(5,0,0)=A\in P$.

For $B=(1,1,1)$, $d(P,B)=|\overrightarrow{AB}|\,|\cos\angle(\overrightarrow{AB},\vec{u})|$ where $\vec{u}=\langle 1,2,3\rangle$.




$$d(P,B)=\frac{|\overrightarrow{AB}\cdot\vec{u}|}{|\vec{u}|}=\frac{|-4\cdot 1+1\cdot 2+1\cdot 3|}{\sqrt{1^2+2^2+3^2}}=\frac{1}{\sqrt{14}}$$

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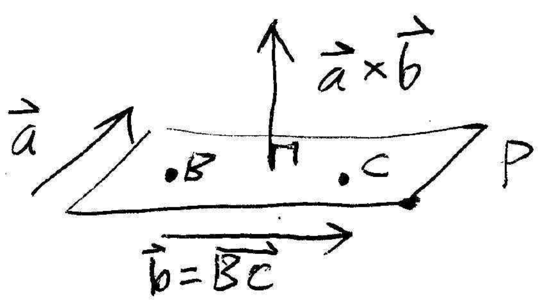
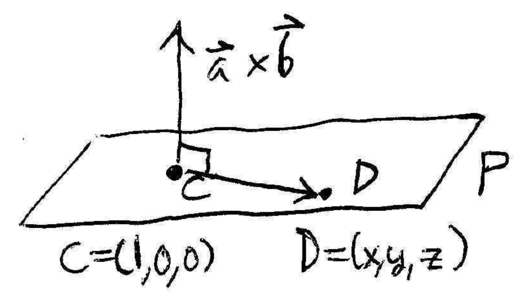
Picture for ①:



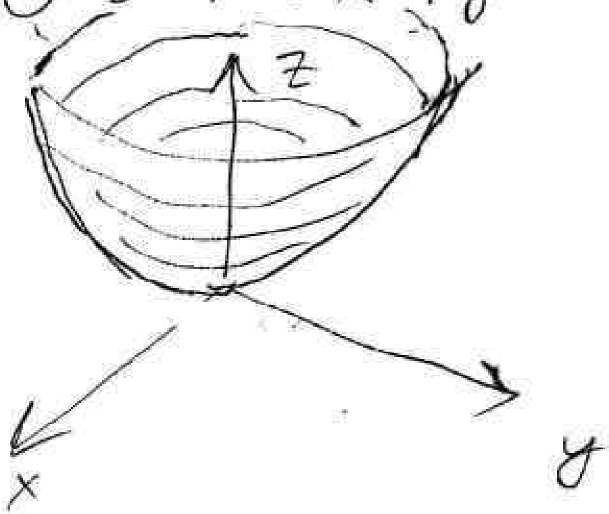



HW10

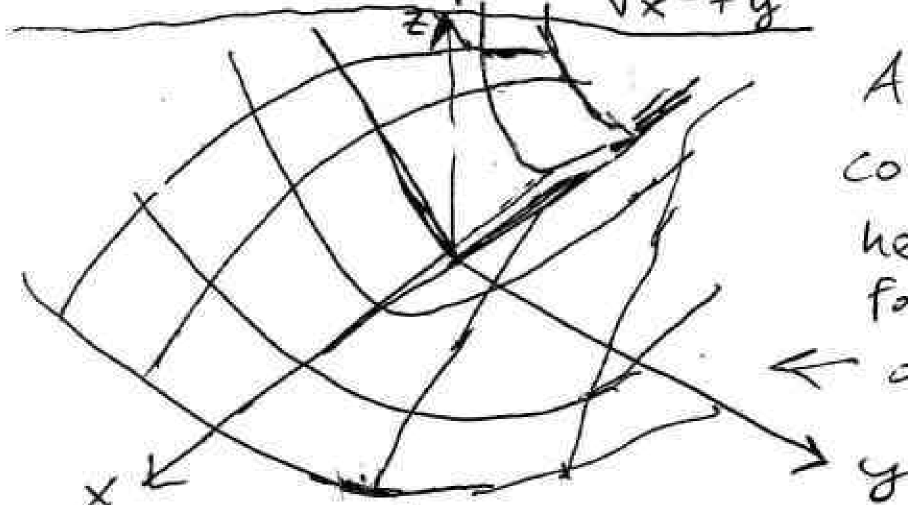
1. ellipsoid
2. elliptical cylinder
3. hyperboloid of one sheet
4. elliptic paraboloid
5. hyperboloid of one sheet
6. elliptic paraboloid

HW11

1. $z = r^2 = x^2 + y^2$

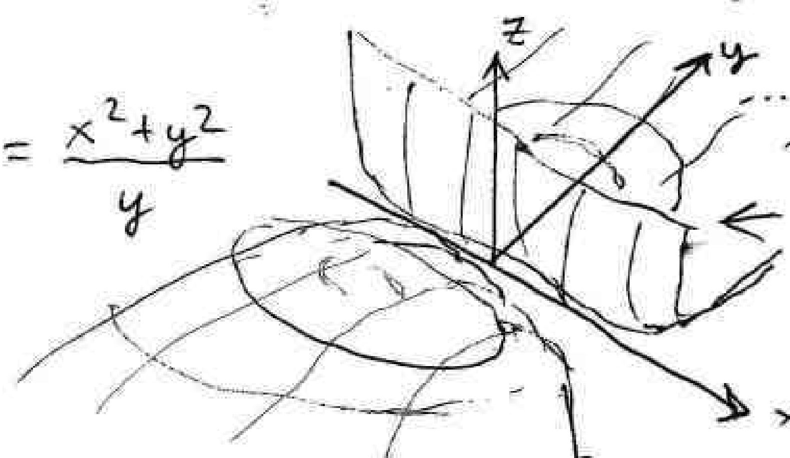



2. $z = r\cos^2\theta = \dfrac{x^2}{r} = \dfrac{x^2}{\sqrt{x^2+y^2}}$




A computer helped for this one...

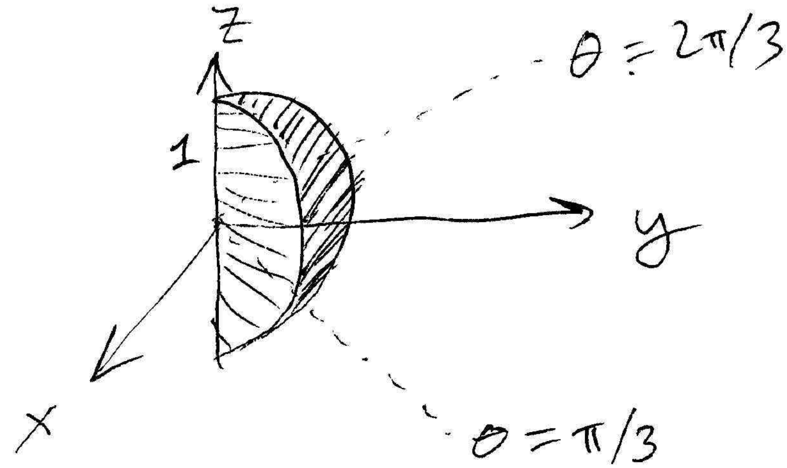
3. $r = z\sin\theta \Leftrightarrow z = \dfrac{r}{\sin\theta} = \dfrac{r^2}{y} = \dfrac{x^2+y^2}{y}$




...and this one.

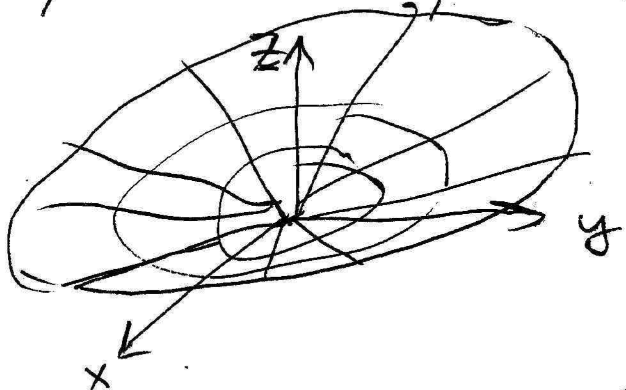
HW12

(1)

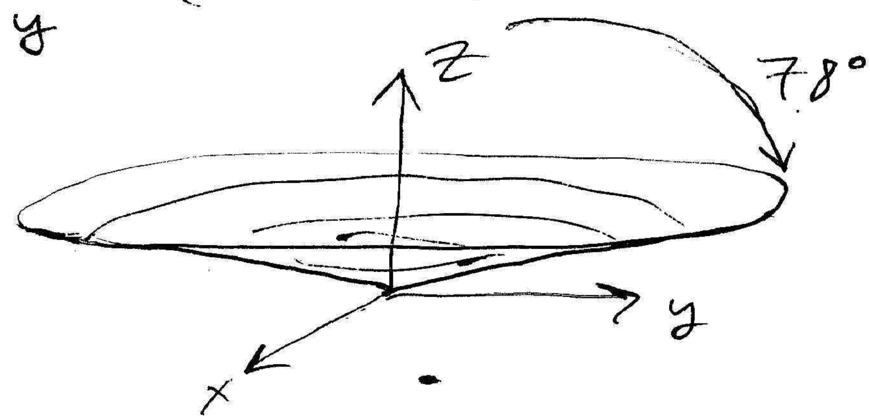



(Like an orange slice.)

(2) $\rho = 5z = 5\rho\cos\phi \iff \cos\phi = \frac{1}{5} \iff \phi \approx 78^\circ$




(A very broad cone.)



HW13 ① $\vec{r} = \langle 1,-2,3\rangle(.6) + (.4)\langle 4,4,4\rangle$ at $t=.4$

$\vec{r} = \langle 1,-2,3\rangle((1-t) + t\langle 4,4,4\rangle$ for $0 \le t \le 1$.

$\frac{d\vec{r}}{dt} = \langle 1,-2,3\rangle(-1) + \langle 4,4,4\rangle = \langle 3,6,1\rangle$.

[$\vec{r}$ at $t=.4$] simplifies to $\langle 2.2, 0.4, 3.4\rangle$.

② $x = 1 + 5\cos t$

$y = 2$ $\quad 0 \le t \le 2\pi$

$z = 4 + 5\sin t$

---

HW14 ① $\vec{r}' = \langle 1, 2t, 3t^2\rangle = \langle 1,1,.75\rangle$ (at $t=.5$)

$\Delta\vec{r} = \langle .503-.5, .503^2-.5^2, .503^3-.5^3\rangle \approx \langle .003, .00301, .00226\rangle$

$d\vec{r} = \langle 1,1,.75\rangle(.003) = \langle .003, .003, .00225\rangle$

$|\vec{r}'| = \sqrt{1^2+1^2+.75^2} \approx 1.601$; $\vec{T} = \vec{r}'/|\vec{r}'| \approx \langle .625, .625, .469\rangle$

② $\int_0^1 \sqrt{1^2+(2t)^2+(3t^2)^2}\,dt \approx 1.863$ (use calculator/computer)

HW16 (1) $\vec{v} = \langle -15\sin(3t), 15\cos(3t), 1\rangle \Rightarrow \vec{T} = \vec{v}/\underbrace{\sqrt{15^2+1^2}}_{|\vec{v}|}$

$\Rightarrow |\vec{T}'| = |\langle -45\cos(3t), -45\sin(3t), 0\rangle| / \sqrt{226} = \frac{45}{\sqrt{226}}$

$\Rightarrow \kappa = |\vec{T}'|/|\vec{v}| = \boxed{45/226} \Rightarrow R = \boxed{226/45} \approx 5.02$

(2) $\vec{v} = \langle 1, 2t, 3t^2\rangle$; $\vec{a} = \langle 0, 2, 6t\rangle$. At $t=1$:

$\vec{v}\times\vec{a} = \langle 1,2,3\rangle \times \langle 0,2,6\rangle = \langle 6, -6, 2\rangle$

$\kappa = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|^3} = \boxed{\sqrt{\frac{76}{14^3}}} \Rightarrow R = \boxed{\sqrt{\frac{14^3}{76}}} \approx 6.01$

HW17: $\vec{v} = \langle 1,0,0\rangle$ at $t=0$. $\vec{a} = \langle 0,2,0\rangle$ at $t=0$.

$\vec{a}_{\parallel} = \text{proj}_{\vec{v}}\vec{a} = \frac{\vec{v}\cdot\vec{a}}{\vec{v}\cdot\vec{v}}\vec{v} = \vec{0}$. $\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = \langle 0,2,0\rangle$.

$\kappa = R^{-1} = |\vec{a}_{\perp}|/|\vec{v}|^2 = 2$. $\vec{T} = \vec{v}/|\vec{v}| = \langle 1,0,0\rangle = \vec{i}$. $R = 2^{-1}$

$\vec{N} = \vec{a}_{\perp}/|\vec{a}_{\perp}| = \langle 0,1,0\rangle = \vec{j}$. $\vec{r} = \langle 0,0,0\rangle = \vec{0}$ at $t=0$.

$\vec{r}_{circle} = \vec{r} + R(\vec{T}\cos\theta + \vec{N}(1+\sin\theta)) = \boxed{\langle 2^{-1}\cos\theta, 2^{-1} + 2^{-1}\sin\theta, 0\rangle}$

HW18 $\frac{d|\vec{v}|}{dt} = \frac{\vec{v}\cdot\vec{a}}{|\vec{v}|}$. $\vec{v} = \langle 1, 2t, 3t^2 \rangle$; $\vec{a} = \langle 0, 2, 6t \rangle$.

At $t=-1$, $\frac{d|\vec{v}|}{dt} = \frac{1\cdot 0 + -2\cdot 2 + 3\cdot -6}{\sqrt{1^2+2^2+3^2}} = \frac{-22}{\sqrt{14}} \approx -5.9$