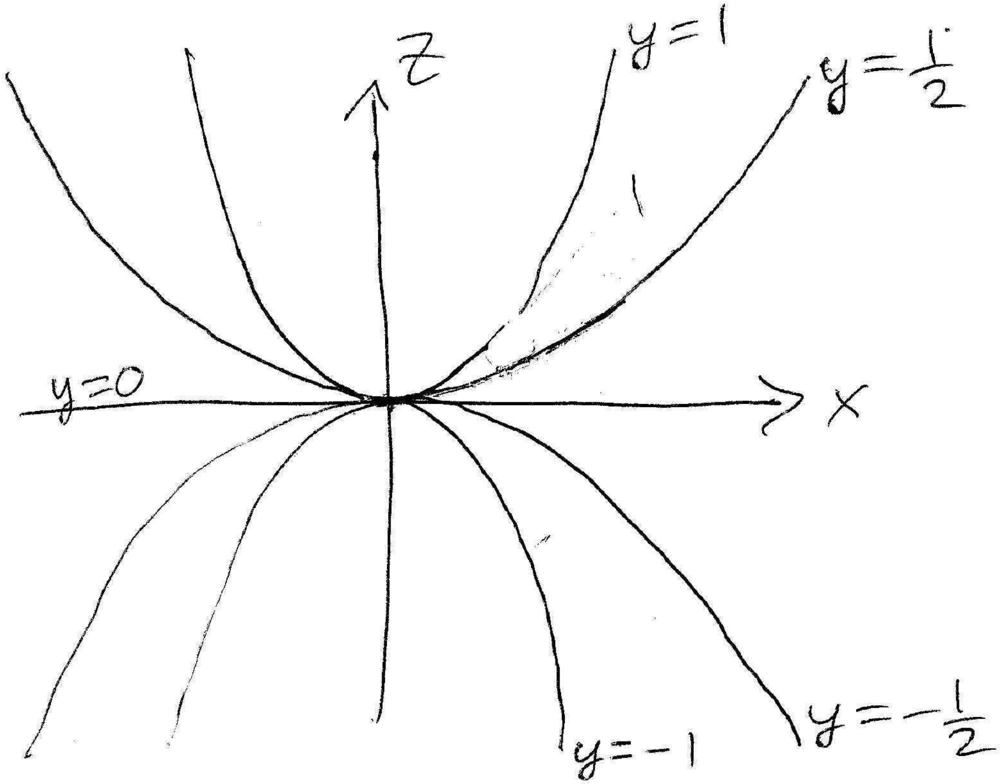
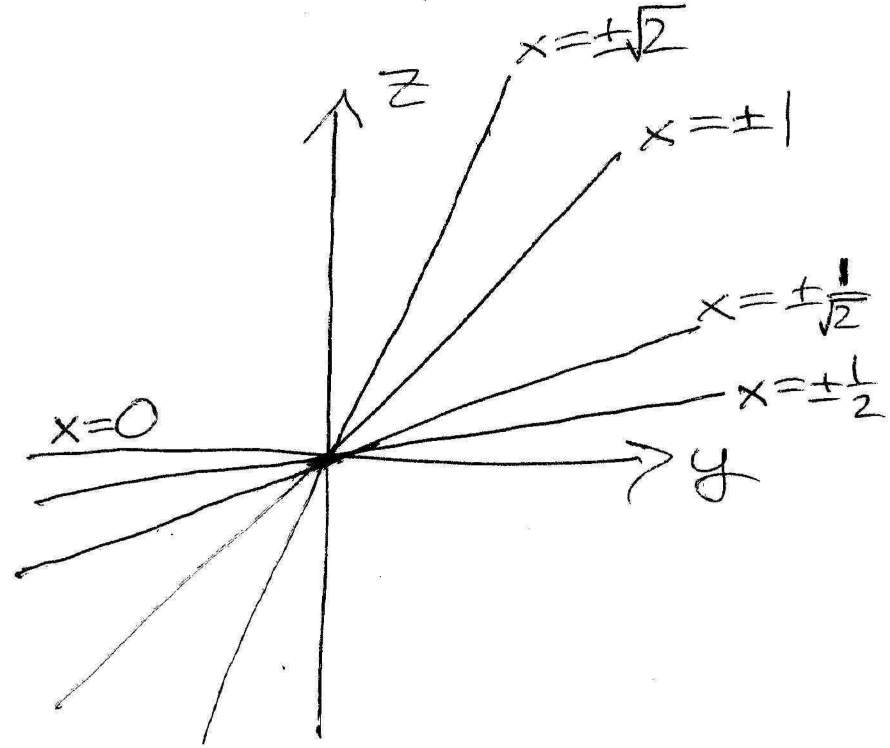
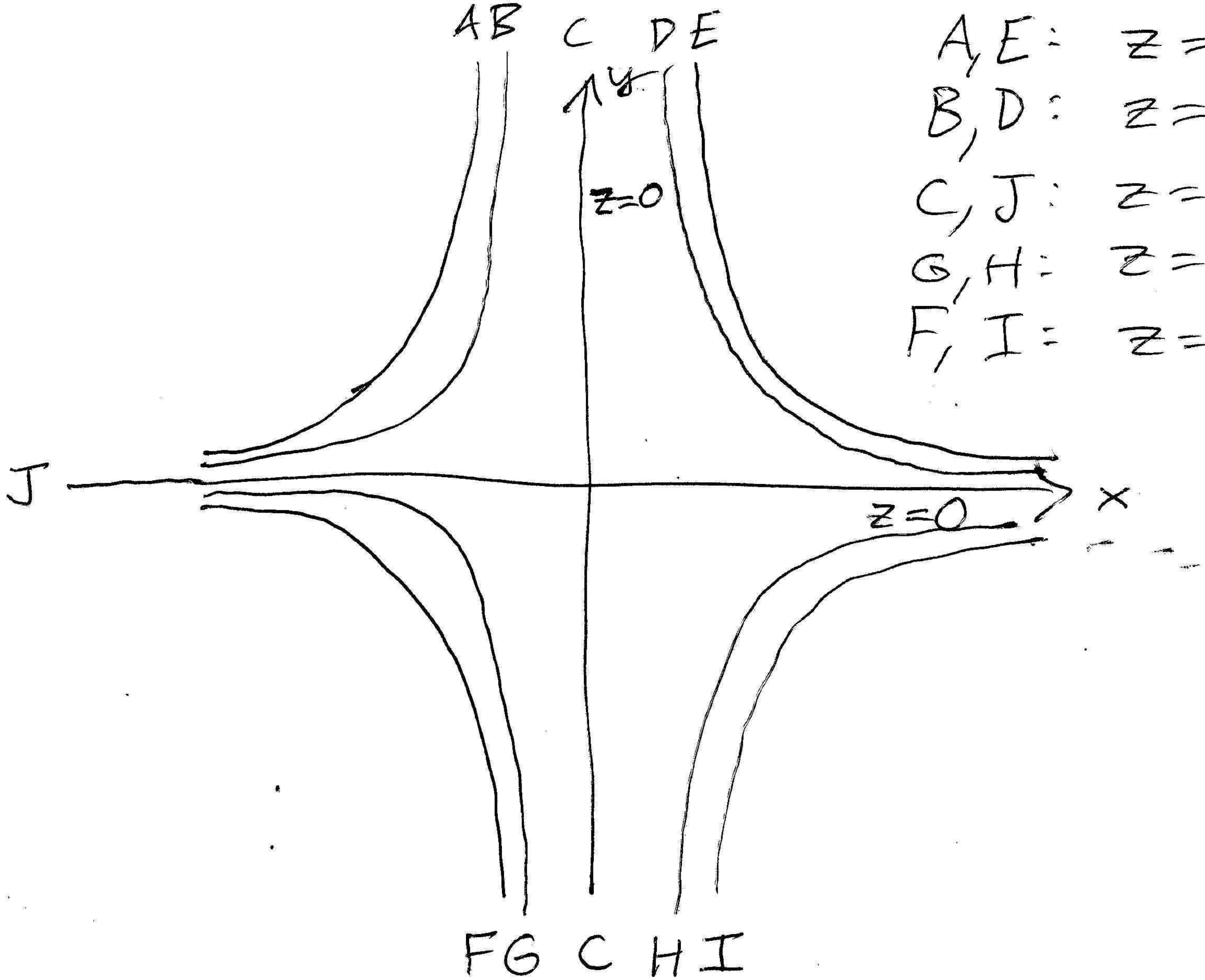




HW19

5 level curves:



A, E: z = 2

B, D: z = 1

C, J: z = 0

G, H: z = −1

F, I: z = −2

HW20 f is essentially discontinuous at $(0,0)$:

$$\lim_{\substack{x \to 0 \\ y=0}} f(x,y) = 0 \quad \& \quad \lim_{\substack{x \to 0 \\ y=x}} f(x,y) = 1/2.$$

g is essentially discontinuous at $(2,3)$:

$$\lim_{\substack{x \to 2 \\ y=3}} g(x,y) = 0 \quad \& \quad \lim_{\substack{x \to 2 \\ y=x+1}} g(x,y) = 1/2.$$

h is essentially discontinuous at $(2,0)$:

$$\lim_{\substack{x \to 2^+ \\ y=x-2}} h(x,y) = \lim_{x \to 2} \frac{x(x-2)}{2(x-2)^2} = \infty.$$

HW21 We want $|f(x,y) - f(4,2)| \le \varepsilon$ to be guaranteed.

$$|f(x,y) - f(4,2)| = |(5x - 7y + 8) - (5\cdot 4 - 7\cdot 2 + 8)|$$
$$= |5(x-4) + (-7)(y-2)|$$
$$\le 5|x-4| + 7|y-2|$$
$$\le 5\delta + 7\delta$$

<u>if</u> $|x-4|, |y-2| \le \delta$. So, any δ where $\delta > 0$ and $12\delta \le \varepsilon$ will do the job.

The simplest tolerance function is thus $\delta = \frac{\varepsilon}{12}$.

HW22 $\quad f_x = -y\sin(xy), \quad f_y = -x\sin(xy)$

$z = \cos(\pi/6) = \frac{\sqrt{3}}{2}$ at $(x,y) = (2, \pi/12)$.

Tangent line slopes:
$$\begin{cases} f_x = -\frac{\pi}{12}\sin\left(\frac{\pi}{6}\right) = -\pi/24 \\ f_y = -2\sin\left(\frac{\pi}{6}\right) = -1 \end{cases}$$

Tangent lines:
$$\begin{cases} z - \sqrt{3}/2 = (-\pi/24)(x-2), \quad y = \pi/12 \\ z - \sqrt{3}/2 = (-1)(y - \pi/12), \quad x = 2 \end{cases}$$

$f(x,y)$	$f_x(x,y)$	$f_y(x,y)$	$f(a,b)$	$f_x(a,b)$	$f_y(a,b)$	a	b
x^2-y^3	$2x$	$-3y^2$	0	16	-48	8	4
x^2y^{-1}	$2xy^{-1}$	$-x^2y^{-2}$	1	$2/3$	$-1/9$	3	9
$\ln(x-y)$	$(x-y)^{-1}$	$-(x-y)^{-1}$	0	1	-1	5	4

HW 23

$f(x,y)$	dx	dy	$f(a,b)+f_x(a,b)\,dx+f_y(a,b)\,dy$	$f(a+dx,b+dy)$
x^2-y^3	0.1	0.03	0.16	0.159173
x^2y^{-1}	0.05	-0.05	$1.03\overline{8}$	$1.039385\ldots$
$\ln(x-y)$	0.3	0.2	0.1	$0.095310\ldots$

tangent plane approx. exact

245 HW26

$$\left.\begin{aligned}\frac{\partial z}{\partial r} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}\\ \frac{\partial z}{\partial \theta} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}\end{aligned}\right\} (1)$$

$$\frac{\partial z}{\partial x} = z_x = \underbrace{ye^{x/y} + xye^{x/y}/y}_{\text{product rule}} = (x+y)e^{x/y}$$

$$\frac{\partial z}{\partial y} = z_y = \overbrace{xe^{x/y} + xye^{x/y}(-x/y^2)} = (x - (x^2/y))e^{x/y}$$

When $r=2$ & $\theta = \pi/6$, $\left\{\begin{aligned} x &= 2\cos \pi/6 = \sqrt{3}\\ y &= 2\sin \pi/6 = 1\end{aligned}\right.$,

which implies (2) $z_x = (\sqrt{3}+1)e^{\sqrt{3}}$ & $z_y = (\sqrt{3}-3)e^{\sqrt{3}}$.

$$(3)\left\{\begin{aligned} \partial x/\partial r &= x_r = \cos\theta = \sqrt{3}/2\\ \partial x/\partial\theta &= x_\theta = -r\sin\theta = -1\\ \partial y/\partial r &= y_r = \sin\theta = 1/2\\ \partial y/\partial\theta &= y_\theta = r\cos\theta \underbrace{= \sqrt{3}}_{\text{at } (2,\pi/6)=(r,\theta)}\end{aligned}\right.$$

Finally, plug in (2) & (3) into (1):

$$\left\{\begin{aligned} z_r &= \sqrt{3}e^{\sqrt{3}}\\ z_\theta &= (2-4\sqrt{3})e^{\sqrt{3}}\end{aligned}\right.$$

#255 $S = 2(xy + yz + zx)$

$$\frac{dS}{dt} = \frac{\partial S}{\partial x}\frac{dx}{dt} + \frac{\partial S}{\partial y}\frac{dy}{dt} + \frac{\partial S}{\partial z}\frac{dz}{dt} \qquad (1)$$

$$\left.\begin{aligned} \partial S/\partial x &= S_x = 2(y+z) = \text{[illegible] in.} \\ \partial S/\partial y &= S_y = 2(z+x) = 6 \text{ in.} \\ \partial S/\partial z &= S_z = 2(x+y) = 10 \text{ in} \end{aligned}\right\} \quad (2)$$

$$dx/dt = dy/dt = dz/dt = 0.5 \text{ in/min} \qquad (3)$$

Plug in (2) & (3) into (1): $\frac{dS}{dt} = 12 \text{ in}^2/\text{min}$

#257

$$\frac{dT}{dt} = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt} \quad (1)$$

$$(T_x, T_y) = (4, 3) \quad \text{at} \quad (x,y) = (2,3).$$

$$\boxed{\begin{aligned} dx/dt &= \tfrac{1}{2}(1+t)^{-1/2} &= 1/4 \\ dy/dt &= 1/3 &= 1/3 \\ x &= \sqrt{1+t} &= 2 \\ y &= 2+t/3 &= 3 \end{aligned}}$$

at $t=3$

$$\left.\begin{matrix} 2 \\ 3 \end{matrix}\right\} \Rightarrow (T_x, T_y) = (4,3) \text{ at } t=3.$$

Plug the #'s into (1): $\frac{dT}{dt} = 2$.

(The units of dT/dt are $°C$ / second.)

HW27

$$f(x,y) = (x^2+y^2+1)^{-1} = 4/9$$

$$f_x = -2x\,(x^2+y^2+1)^{-2} = -16/81$$

$$f_y = -2y\,(x^2+y^2+1)^{-2} = -32/81$$

At $(x,y) = (\frac{1}{2}, 1)$

Line tangent to $\frac{4}{9} = f(x,y)$ at $(x,y) = (1/2, 1)$:

$$0 = (\nabla f) \cdot \langle x - 1/2, y - 1 \rangle \quad \text{where}$$

$$\boxed{\vec{\nabla} f = -(16/81)\langle 1, 2 \rangle} \quad (\text{at } (1/2, 1)).$$

Simplified line: $\boxed{0 = 2x + 4y - 5}$

$\vec{v}$ (examples)	$\langle 1,0 \rangle$	$\langle 0,-1 \rangle$	$\langle \frac{3}{5}, -\frac{4}{5} \rangle$	$\langle \frac{5}{13}, \frac{12}{13} \rangle$	$\langle \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$
$(D_v f)(1/2, 1)$	$-\frac{16}{81}$	$\frac{32}{81}$	$\frac{16}{81}$	$-\frac{16}{81} \cdot \frac{29}{13}$	$\frac{8\sqrt{2}}{27}$

② At $(x,y,z)=(2,1,1)$, the plane HW28

tangent to $34 = f(x,y,z) = 2x^4+y^4+z^4$

is given by $0 = (\vec{\nabla} f)(2,1,1) \cdot \langle x-2, y-1, z-1\rangle$

where $\vec{\nabla} f = \langle 8x^3, 4y^3, 4z^3\rangle = \langle 64, 4, 4\rangle$

at $(2,1,1)$. So, the plane is given by

$0 = 64(x-2) + 4(y-1) + 4(z-1)$.

(This simplifies to $34 = 16x + y + z$.)

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① A plot suggests $(x,y)=(2,1)$ is not a terrible place to start. 30 steps from there using $\varepsilon = 0.05$ yields $(x,y) = (2.506627, 0.626669)$.

HW28 #1 solution

n	x_n	y_n	f	f_x	f_y	epsilon
0	2.000000	1.000000	0.255654	2.611063	-0.832294	0.05
1	2.130553	0.958385	0.718803	3.762428	-0.966977	0.05
2	2.318675	0.910036	1.474241	3.185138	-1.190686	0.05
3	2.477931	0.850502	1.849193	0.271620	-1.266951	0.05
4	2.491512	0.787155	1.921901	0.076549	-0.948187	0.05
5	2.495340	0.739745	1.960799	0.080683	-0.677889	0.05
6	2.499374	0.705851	1.980699	0.045833	-0.480344	0.05
7	2.501666	0.681834	1.990603	0.032623	-0.336510	0.05
8	2.503297	0.665008	1.995454	0.021193	-0.234755	0.05
9	2.504357	0.653270	1.997809	0.014437	-0.163232	0.05
10	2.505078	0.645109	1.998946	0.009735	-0.113320	0.05
11	2.505565	0.639443	1.999494	0.006648	-0.078584	0.05
12	2.505897	0.635514	1.999757	0.004546	-0.054462	0.05
13	2.506125	0.632790	1.999883	0.003123	-0.037729	0.05
14	2.506281	0.630904	1.999944	0.002149	-0.026130	0.05
15	2.506388	0.629597	1.999973	0.001482	-0.018094	0.05
16	2.506463	0.628693	1.999987	0.001023	-0.012528	0.05
17	2.506514	0.628066	1.999994	0.000707	-0.008674	0.05
18	2.506549	0.627633	1.999997	0.000489	-0.006005	0.05
19	2.506573	0.627332	1.999999	0.000338	-0.004157	0.05
20	2.506590	0.627125	1.999999	0.000234	-0.002878	0.05
21	2.506602	0.626981	2.000000	0.000162	-0.001992	0.05
22	2.506610	0.626881	2.000000	0.000112	-0.001379	0.05
23	2.506616	0.626812	2.000000	0.000077	-0.000954	0.05
24	2.506620	0.626764	2.000000	0.000054	-0.000661	0.05
25	2.506622	0.626731	2.000000	0.000037	-0.000457	0.05
26	2.506624	0.626708	2.000000	0.000026	-0.000317	0.05
27	2.506625	0.626693	2.000000	0.000018	-0.000219	0.05
28	2.506626	0.626682	2.000000	0.000012	-0.000152	0.05
29	2.506627	0.626674	2.000000	0.000009	-0.000105	0.05
30	2.506627	0.626669	2.000000	0.000006	-0.000073	0.05

It is actually possible to find a formula describing the infinitely many critical points of $\sin(xy)+\cos(x^2)$:

$(x,y)=(0,0)$ or $\pm\left(\sqrt{n\pi},\ (m+\frac{1}{2})\sqrt{\pi/n}\right)$

where m & n are integers & $n>0$.

At $(m,n)=(0,2)$, $(x,y)\approx\pm(2.506627,\ 0.626657)$.

Pretty close!

HW29

Solve $\begin{cases} 0 = f_x = 3x^2 - 2x - 10y & (1) \\ 0 = f_y = 3y^2 - 10x & (2) \end{cases}$

$(2) \Rightarrow x = (3/10)y^2 \underset{(1)}{\Longrightarrow} 0 = \frac{27}{100}y^4 - \frac{3}{5}y^2 - 10y \quad (3)$

$(3) \Leftrightarrow y = 0$ or $27y^3 - 60y - 1000 = 0. \quad (4)$

$(4) \Leftrightarrow y = 0$ or $y \approx 3.5552 \quad (5)$

$(5) \underset{(2)}{\Longrightarrow} (x, y) = (0,0)$ or $(x,y) \approx (3.7918, 3.5552)$

critical points

$f_{xx} f_{yy} = (6x - 2)(6y)$ & $f_{xy}^2 = (-10)^2 = 100$

$(0,0)$ is a saddle point because there $f_{xx} f_{yy} = 0 < f_{xy}^2$.

At the other critical point, $f_{xx} \approx 20.75 > 0$ & $f_{xx} f_{yy} \approx 443 > f_{xy}^2$, so this point is the location of a local minimum

$\approx f(3.5552, 3.7918) \approx -48.73$.