

① If $x = \sin \varphi \cos \theta$ & $y = \sin \varphi \sin \theta$, HW24

and, at $(\varphi, \theta) = (\pi/6, \pi/3)$, $z = f(x, y) = g(\varphi, \theta)$

satisfies $\frac{\partial z}{\partial x} = 3$, $\frac{\partial z}{\partial y} = 4$, then $\frac{\partial z}{\partial \varphi}$, $\frac{\partial z}{\partial \theta} = ?$, $?$

② If instead $\frac{\partial z}{\partial \varphi} = 3$, $\frac{\partial z}{\partial \theta} = 4$, then $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y} = ?$, $?$

(Still at $(\varphi, \theta) = (\pi/6, \pi/3)$.)

③ If $z = f(x, y)$, $x = t \cos(3t)$, $y = g(t)$,

and, at $t = \frac{\pi}{12}$, $\frac{\partial z}{\partial y} = -1$ and $\frac{dy}{dt} = 7 = \frac{dz}{dt}$,

then $\frac{\partial z}{\partial x} = ?$