f(x,y)	-0.1	-0.01	-0.001	0	0.001	0.01	0.1	x
0.1	0.05000	0.00099	0.00001	0.00000	0.00001	0.00099	0.05000	
0.01	0.00990	0.00500	0.00010	0.00000	0.00010	0.00500	0.00990	
0.001	0.00100	0.00099	0.00050	0.00000	0.00050	0.00099	0.00100	
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	
-0.001	-0.00100	-0.00099	-0.00050	0.00000	-0.00050	-0.00099	-0.00100	
-0.01	-0.00990	-0.00500	-0.00010	0.00000	-0.00010	-0.00500	-0.00990	
-0.1	-0.05000	-0.00099	-0.00001	0.00000	-0.00001	-0.00099	-0.05000	
y								

g(x,y)	0.9	0.99	0.999	1	1.001	1.01	1.1	x
0.1	4.050	9.704	9.979	10.000	10.019	10.100	6.050	
0.01	0.802	49.005	98.812	100.000	99.208	51.005	1.198	
0.001	0.081	9.704	499.001	1000.000	501.001	10.100	0.121	
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
-0.001	-0.081	-9.704	-499.001	-1000.000	-501.001	-10.100	-0.121	
-0.01	-0.802	-49.005	-98.812	-100.000	-99.208	-51.005	-1.198	
-0.1	-4.050	-9.704	-9.979	-10.000	-10.019	-10.100	-6.050	
y								

h(x,y)	-0.1	-0.01	-0.001	0	0.001	0.01	0.1	x
0.1	0.3536	0.0099	0.0001	0.0000	0.0001	0.0099	0.3536	
0.01	0.0985	0.3536	0.0099	0.0000	0.0099	0.3536	0.0985	
0.001	0.0100	0.0985	0.3536	0.0000	0.3536	0.0985	0.0100	
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
-0.001	-0.0100	-0.0985	-0.3536	0.0000	-0.3536	-0.0985	-0.0100	
-0.01	-0.0985	-0.3536	-0.0099	0.0000	-0.0099	-0.3536	-0.0985	
-0.1	-0.3536	-0.0099	-0.0001	0.0000	-0.0001	-0.0099	-0.3536	
y								

k(x,y)	-0.1	-0.01	-0.001	0	0.001	0.01	0.1	x
0.1	0.7943	0.6310	0.5012	0.0000	0.5012	0.6310	0.7943	
0.01	0.9772	0.9550	0.9333	0.0000	0.9333	0.9550	0.9772	
0.001	0.9977	0.9954	0.9931	0.0000	0.9931	0.9954	0.9977	
0	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	
-0.001	0.9977	0.9954	0.9931	0.0000	0.9931	0.9954	0.9977	
-0.01	0.9772	0.9550	0.9333	0.0000	0.9333	0.9550	0.9772	
-0.1	0.7943	0.6310	0.5012	0.0000	0.5012	0.6310	0.7943	
y								

HW19

① continuous ③ discontinuous

② discontinuous ④ discontinuous (See tables...)

HW20

① $|f(4+dx, 3+dy)-f(4,3)|$

$= |(8(4+dx)-5(3+dy)+1)-(8\cdot 4-5\cdot 3+1)|$

$= |8\,dx-5dy| \le 8|dx|+5|dy| < 13\delta$ (if $|dx|, |dy| < \delta$)

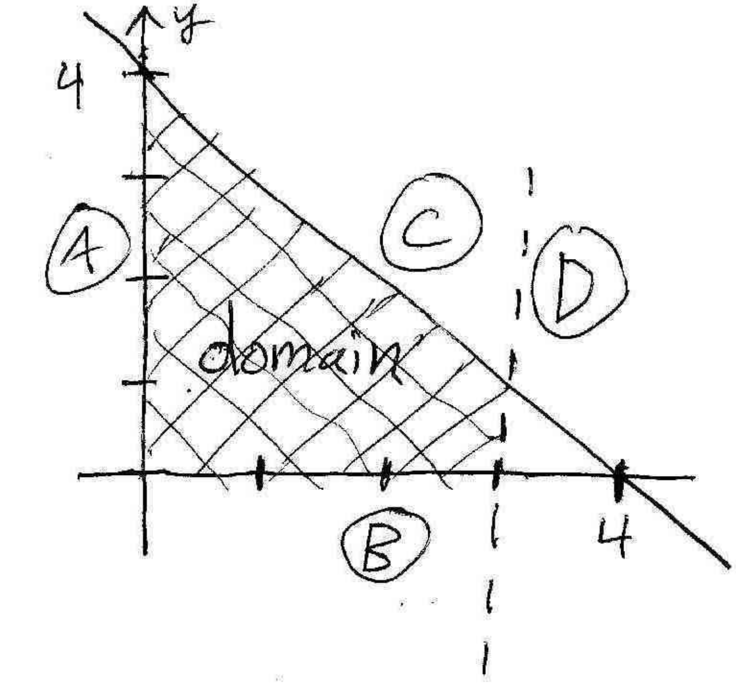
So, any $\delta > 0$ with $13\delta \le \varepsilon$ will work. For example: $\boxed{\delta = \varepsilon/13}$.

② We need $\underset{A}{x \ge 0}$, $\underset{B}{y \ge 0}$, $\underset{C}{4-x-y \ge 0}$, and $\underset{D}{3-x > 0}$

(and we need x, y, & $4-x-y$ not all 0, but $x=0=y \Rightarrow 4-x-y \ne 0$).

Changing the inequalities in A, B, C, D to equalities, we get 4 boundary lines:

$x=0$, $y=0$, $4-x=y$, $3=x$.



HW20

3 $\quad \varepsilon = 0.01$

$a = 0.074$

$\delta = \min\{.037,\ .00002738\}$

$= 2.738 \times 10^{-5}$

Any $|dx| < 2.738 \times 10^{-5}$ works.

$$\left.\begin{aligned} \frac{1}{.074} &= 13.51\underline{3513513513}\ldots \\ \frac{1}{.074-\delta} &= 13.51\underline{85153364198}\ldots \end{aligned}\right\}$$

The difference is $\approx .005$, $\leq \varepsilon$ as it should be.

(You might think δ could be twice as big since the difference was about $\varepsilon/2$. This is almost true. If you double δ, the difference becomes a tiny bit ($\approx 7 \times 10^{-6}$) more than ε.)

HW20 ③ Comment: Tolerance formulas (δ) just need to be small enough.

$\delta = \min\left\{\frac{|a|}{2}, \frac{a^2\varepsilon}{2}\right\}$ is relatively simple & small enough to guarantee $|\text{error}| < \varepsilon$. The biggest tolerance that works (for $h(x) = \frac{1}{x}$ at $x=a$) is $\delta = \min\left\{|a|, \frac{a^2\varepsilon}{1+\varepsilon|a|}\right\}$, but this is a bit more complicated.

HW21

① $f(x,y) = \ln(x^2 - 3y^2) = \ln 1 = 0$ @ $(2,1)$

$f_x(x,y) = 2x(x^2-3y^2)^{-1} = 4/1 = 4$ @ $(2,1)$

$f_y(x,y) = -6y(x^2-3y^2)^{-1} = {}^{-6}/_1 = -6$ @ $(2,1)$

$f(2+.02, 1) \approx f(2,1) + f_x(2,1)(.02) = 0.08$

$f(2+(-.01), 1) \approx f(2,1) + f_x(2,1)(-.01) = -0.04$

$f(2, 1+.05) \approx f(2,1) + f_y(2,1)(.05) = -0.3$

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② $3x^2 + 2xy + 1y^2 + 0 = 3x^2 + 2xy + y^2$

③ $\left(e^{(xy^{-1})}\right)(-xy^{-2}) = -xy^{-2}e^{x/y}$

(I like using negative exponents to avoid the quotient rule when the numerator is constant.)

HW22

(1) $f(x,y)=\frac{x+y}{x-y}=\frac{5+4.5}{5-4.5}=19$ @ $(5,4.5)$

$f_x=\frac{(1+0)(x-y)-(x+y)(1-0)}{(x-y)^2}=\frac{-2y}{(x-y)^2}=-36$ @ $(5,4.5)$

$f_y=\frac{(0+1)(x-y)-(x+y)(0-1)}{(x-y)^2}=\frac{2x}{(x-y)^2}=40$ @ $(5,4.5)$

$f(5+.01,4.5+(-.01))\approx 19+(-36)(.01)+(40)(-.01)$

$=\boxed{18.24}$

(The exact value is $18.\overline{2692307}$.)

(2)

$f(x,y)=x^{\frac{1}{3}}y^{-\frac{1}{3}}=1$ @ $(1000,1000)$

$f_x=\frac{1}{3}x^{-2/3}y^{-1/3}=1/3000$ @ $(1000,1000)$

$f_y=x^{1/3}\cdot(-1/3)y^{-4/3}=-1/3000$ @ $(1000,1000)$

$f(1000+15,1000+6)\approx 1+\frac{15}{3000}-\frac{6}{3000}=1+\frac{3}{1000}=\boxed{1.003}$