

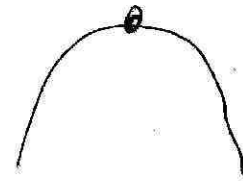
Background info:

① $y = Ax^2 + Bx + C$



local min

local max

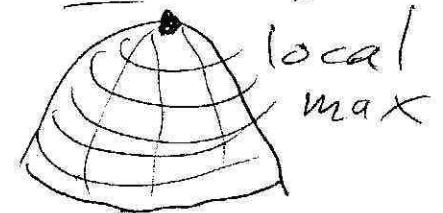


$A < 0$

$$y=0 \iff x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

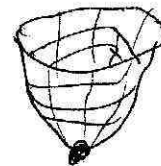
$y=0$ has real solutions $\iff 4AC - B^2 < 0$

② $z = Ax^2 + Bxy + Cy^2$



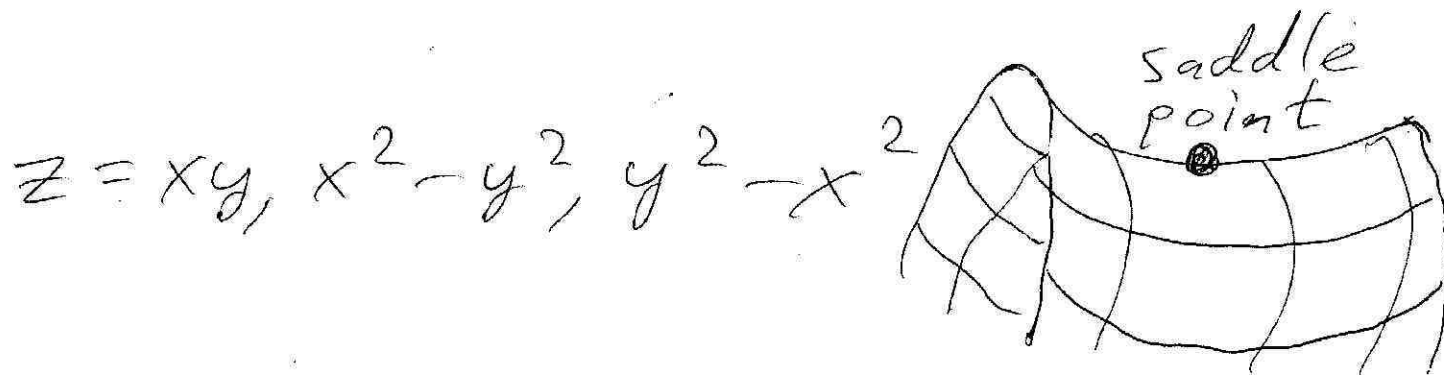
local max

Simple cases: $z = x^2 + y^2$



local min

$$z = -x^2 - y^2$$



saddle point

③ $z = Ax^2 + Bxy + Cy^2$ general case:

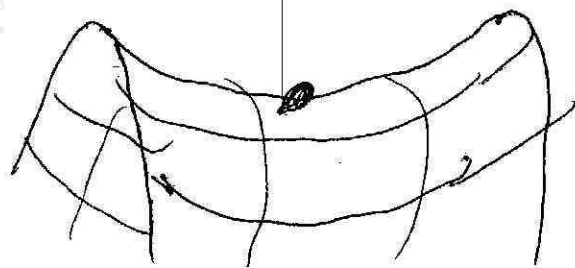
$$D = 4AC - B^2$$

$$u = x + \frac{By}{2A}$$

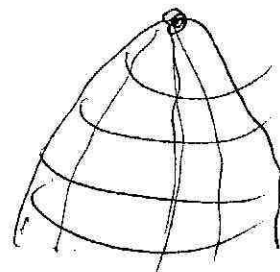
$$v = \frac{y}{2A}$$

$$z = A(u^2 + Dv^2)$$

$D < 0 \Rightarrow$ saddle point



$D > 0$ & $A < 0 \Rightarrow$ local max



$D > 0$ & $A > 0 \Rightarrow$ local min

