

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

$$C(A) = \left\langle \left\{ \begin{bmatrix} 1 \\ 5 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 11 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 12 \end{bmatrix} \right\} \right\rangle$$

$$N(A) = \{ \vec{x} \mid A\vec{x} = \vec{0} \}$$

$$R(A) = C(A^t) = \left\langle \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 9 \\ 10 \\ 11 \\ 12 \end{bmatrix} \right\} \right\rangle$$

$$L(A) = \{ \vec{y} \mid \vec{y}^T A = \vec{0} \} = N(A^t)$$

$$A^t = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$$

$$A \xrightarrow{\text{RREF}} \begin{bmatrix} \textcircled{1} & 0 & -1 & -2 \\ 0 & \textcircled{1} & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ tells us:}$$

$$\left\{ \begin{bmatrix} 1 \\ 5 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 10 \end{bmatrix} \right\} \text{ is indep. \& spans } C(A)$$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \right\} \text{ is indep. \& spans } R(A)$$

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ is indep. \& spans } N(A).$$

$$A^t \xrightarrow{\text{RREF}} \begin{bmatrix} \textcircled{1} & 0 & -1 \\ 0 & \textcircled{1} & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ tells us:}$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \right\} \text{ is indep. \& spans } \overbrace{R(A)}^{C(A^t)}$$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\} \text{ is indep. \& spans } \overbrace{C(A)}^{R(A^t)}$$

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\} \text{ is indep. \& spans } \overbrace{L(A)}^{N(A^t)}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 \\ 5 & 6 & 7 & 8 & | & 0 & 1 & 0 \\ 9 & 10 & 11 & 12 & | & 0 & 0 & 1 \end{bmatrix} = [A | I_3] \quad \text{RREF}$$

$$\text{RREF} \rightarrow \begin{bmatrix} \textcircled{1} & 0 & -1 & -2 & | & 0 & -2.5 & 1.5 \\ 0 & \textcircled{1} & 2 & 3 & | & 0 & 2.25 & -1.25 \\ 0 & 0 & 0 & 0 & | & 1 & -2 & 1 \end{bmatrix}$$

tells us:

This is the RREF of A . $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ is indep. & spans $L(A)$

It tells us about $C(A)$, $R(A)$, $N(A)$ like before.

$$\left[\begin{array}{ccc|cccc} 1 & 5 & 9 & 1 & 0 & 0 & 0 \\ 2 & 6 & 10 & 0 & 1 & 0 & 0 \\ 3 & 7 & 11 & 0 & 0 & 1 & 0 \\ 4 & 8 & 12 & 0 & 0 & 0 & 1 \end{array} \right] = [A^t \mid I_4]$$

$$\text{RREF} \rightarrow \left[\begin{array}{ccc|cccc} \textcircled{1} & 0 & -1 & 0 & 0 & -2 & 1.75 \\ 0 & \textcircled{1} & 2 & 0 & 0 & 1 & -0.75 \\ 0 & 0 & 0 & 1 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{array} \right]$$

This is the RREF
of A^t . It tells
us about
 $R(A)$, $C(A)$, $L(A)$
like before.

$\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ is indep.
& spans $N(A)$.