① A is row-equivalent to Day 7

$$R = \begin{array}{c} \begin{bmatrix} 1 & 8 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccccc} P & F & P & P & F & F \end{array} \end{array}$$

F = free P = pivot

Therefore, columns $\underline{1}$, $\underline{3}$, and $\underline{4}$ of A are linearly independent and have the same $\underline{\text{span}}$ as all the columns of A. Also, the 2nd column of A equals $\underline{8}$ times the 1st column of A. We also see that columns $\underline{3}$ and $\underline{4}$ of A add to equal column $\underline{6}$.

Day 7

② $\left\langle \begin{bmatrix} 1 \\ 5i \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\rangle = ?$

③ Manually perform Gram-Schmidt to find orthogonal vectors with same span as

$$\begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \\ 10 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \end{bmatrix}.$$

(Compare to examples in section O.)

④ Find orthonormal vectors with same span as the orthogonal vectors

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}.$$

Day 7

(2) $\overline{1} \cdot 2 + \overline{5i} \cdot 1 = 1 \cdot 2 - 5i \cdot 1 = 2 - 5i$

(3) $\vec{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \vec{w}_1$; $\vec{v}_2 = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 10 \end{bmatrix}$; $\vec{w}_2 = \vec{v}_2 - \frac{\langle \vec{w}_1, \vec{v}_2 \rangle}{\langle \vec{w}_1, \vec{w}_1 \rangle} \vec{w}_1$

$\vec{w}_2 = \vec{v}_2 - \frac{10}{5} \vec{w}_1 = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 10 \end{bmatrix} - \begin{bmatrix} 4 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 0 \\ 8 \end{bmatrix}$; $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \end{bmatrix}$;

$\vec{w}_3 = \vec{v}_3 - \frac{\langle \vec{w}_1, \vec{v}_3 \rangle}{\langle \vec{w}_1, \vec{w}_1 \rangle} \vec{w}_1 - \frac{\langle \vec{w}_2, \vec{v}_3 \rangle}{\langle \vec{w}_2, \vec{w}_2 \rangle} \vec{w}_2 = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \end{bmatrix} - \frac{1}{5} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \frac{8}{89} \begin{bmatrix} -4 \\ 3 \\ 0 \\ 8 \end{bmatrix}$

Final answer: $\begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 32/89 - 2/5 \\ -24/89 \\ 4 \\ 1 - 1/5 - 64/89 \end{bmatrix}$

(4) $\begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}$

Day 7

(5) Let $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Which of the following vectors are orthogonal both to $\vec{u}$ and to $\vec{v}$?

$$\underset{\text{not } \perp \vec{v}}{\begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}}, \quad \underset{\text{not } \perp \vec{v}}{\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}}, \quad \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \quad \underset{\text{not } \perp \vec{v}}{\begin{bmatrix} 5 \\ -1 \\ -1 \end{bmatrix}}, \quad \underset{\text{not } \perp \vec{u}}{\begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}}$$

(6) Sort these vectors from least norm to greatest norm:

$$\begin{bmatrix} 3 \\ 2+i \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2-2i \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} -3i \\ -3 \end{bmatrix}$$

(See the solution two pages down.)

Day 8

(1) The Pauli matrices describe what happens when you try to measure which way an electron "spins" using a magnet (and some other equipment): $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, and $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Match each matrix on the left with an equal matrix on the right:

a) $XY - YX$ — α) YY^*

b) ZX — β) $2iZ$

c) $-iXYZ$ — γ) $XZ + 2iY$

d) $YZ + ZY$ — δ) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(You should just compute the eight matrices to see what matches what.)

6 $\left\|\begin{bmatrix}3\\2+1i\end{bmatrix}\right\| = \sqrt{3^2+2^2+1^2} = \sqrt{14}$ $\quad$ $\left\|\begin{bmatrix}0\\4\end{bmatrix}\right\| = \sqrt{4^2} = \sqrt{16}$

$\left\|\begin{bmatrix}3\\2-2i\end{bmatrix}\right\| = \sqrt{3^2+2^2+2^2} = \sqrt{17}$ $\quad$ $\left\|\begin{bmatrix}-3i\\-3\end{bmatrix}\right\| = \sqrt{3^2+3^2} = \sqrt{18}$

$\boxed{\text{Day 7}}$

$\begin{bmatrix}3\\2+i\end{bmatrix}, \begin{bmatrix}0\\4\end{bmatrix}, \begin{bmatrix}3\\2-2i\end{bmatrix}, \begin{bmatrix}-3i\\-3\end{bmatrix}$

$\boxed{\text{Day 8}}$

1

$X = \begin{bmatrix}0&1\\1&0\end{bmatrix}$, $Y = \begin{bmatrix}0&-i\\i&0\end{bmatrix}$, $Z = \begin{bmatrix}1&0\\0&-1\end{bmatrix}$.

$Y^* = \overline{Y^t} = Y$

$XY - YX = \begin{bmatrix}i&0\\0&-i\end{bmatrix} - \begin{bmatrix}-i&0\\0&i\end{bmatrix} = 2iZ$;

$ZX = \begin{bmatrix}0&1\\-1&0\end{bmatrix}$; $\quad YY^* = \begin{bmatrix}0&-i\\i&0\end{bmatrix}\begin{bmatrix}0&-i\\i&0\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$;

$YZ + ZY = \begin{bmatrix}0&i\\i&0\end{bmatrix} + \begin{bmatrix}0&-i\\-i&0\end{bmatrix} = \begin{bmatrix}0&0\\0&0\end{bmatrix}$;

$XZ + 2iY = \begin{bmatrix}0&-1\\1&0\end{bmatrix} + \begin{bmatrix}0&2\\-2&0\end{bmatrix} = ZX$

$(-iX)(YZ) = \begin{bmatrix}0&-i\\-i&0\end{bmatrix}\begin{bmatrix}0&i\\i&0\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix} = YY^*$

Day 8

(2) Compute $\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$ by hand.

Repeat for $\begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$.

(3) If $\left\langle \begin{bmatrix} 3 \\ 4 \end{bmatrix}, A \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} \right\rangle = 8$ is well defined, then how many rows has A? how many columns?

(4) This is wrong: "If A and B are 2×2 matrices, then $(A+B)^2$ equals $A^2 + 2AB + B^2$." Actually, $(A+B)^2 = A^2 + \underline{AB + BA + B^2}$.

Day 8

2 $\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix}$

$3 \times 1 \quad 1 \times 3 \quad 3 \times 3$

$\begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \end{bmatrix}$

$1 \times 3 \quad 3 \times 1 \quad 1 \times 1$

3 $\begin{bmatrix} 3 & 4 \end{bmatrix} A \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 8 \end{bmatrix}$

$1 \times 2 \quad 2 \times 3 \quad 3 \times 1 \quad 1 \times 1$

$\boxed{\text{A has 2 rows \& 3 columns.}}$

Day 9

① Give an example of 2×2 invertible matrices A, B such that $A+B$ is not invertible <u>and</u> $AB \neq A^{-1}B^{-1}$.

② Give an example of a 2×2 matrix A such that $A^{-1} = A^* \neq I_2$.

③ There is a matrix $E = \begin{bmatrix} 1 & ? \\ ? & 1 \end{bmatrix}$ such that $E\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ adds five copies of $[a\ \ b]$ to $[c\ \ d]$ to make $\begin{bmatrix} a & b \\ 5a+c & 5b+d \end{bmatrix}$. Find E and then find E^{-1}.

④ If $A\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ and $A\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$, is A invertible? Yes, no, or not enough information?

Day 9

① $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = A$ & $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix} = B$ (Guess)

Let's see if this works: $A+B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ is not invertible because of the $\vec{0}$ column. ✓

$AB = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$; $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$; $B^{-1} = \begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$

$A^{-1}B^{-1} = \begin{bmatrix} -1 & -4 \\ 0 & 1 \end{bmatrix} \neq AB$. ✓ (This was my 2nd try.)

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② $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^{-1} = A^{*} = A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \neq I_2$

---

③ $E = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ $E^{-1} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$

---

④ No: $A\left(\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \vec{0} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in N(A)$.

Day 10

① $D = \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ has orthogonal columns,

but D is not unitary because its columns

do not have <u>norm 1</u>.

② Give an example of a 2×2 unitary matrix U such that U, U^2, U^3, U^4 all $\neq I_2$

but $U^5 = I_2$.

③ Find $\begin{bmatrix} 1 & 3 \\ 1 & 7 \end{bmatrix}^{-1}$ without a calculator.

④ $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$ (compute it!)

is NOT invertible because its columns

are <u>multiples of each other</u>.

or: linearly dependent

Day 10

(2) Rotate $\frac{1}{5}$ of a revolution:

$$\begin{bmatrix} \cos 2\pi/5 & -\sin 2\pi/5 \\ \sin 2\pi/5 & \cos 2\pi/5 \end{bmatrix}$$

(3) $$\frac{1}{1\cdot 7 - 3\cdot 1}\begin{bmatrix} 7 & -3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 7/4 & -3/4 \\ -1/4 & 1/4 \end{bmatrix}$$

(4) $$\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 15 \\ 1 & 2 & 5 \\ 4 & 8 & 20 \end{bmatrix}$$

(5) $$((\bar{A}B)^t C)^{-1} = (B^t \bar{A}^t C)^{-1}$$

$$= (B^t A^* C)^{-1} = C^{-1}(A^*)^{-1}(B^t)^{-1}$$

$$= C^{-1}(A^{-1})^*(B^{-1})^t$$

Day 10

5. $((\overline{A}B)^t C)^{-1}$ equals $\underline{C^{-1}(A^{-1})^*(B^{-1})^t}$.

Select the answer from below.

$\overline{(A^{-1})}(B^{-1})^t C^{-1}$, $\quad C^{-1}(B^{-1})^t(A^{-1})^t$,

$(A^{-1})^*(B^{-1})^t C^{-1}$, $\quad C^{-1}(B^{-1})^t(A^{-1})^*$,

$\boxed{C^{-1}(A^{-1})^*(B^{-1})^t}$, $\quad C^{-1}\overline{(A^{-1})}(B^{-1})^t$.

(Exactly one of the six choices is correct.)

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6. Is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ unitary, self-adjoint, both, or neither? $\underline{\text{Both.}}$

Day 12

① Find a unitary 2×2 whose first column is a positive multiple of $\begin{bmatrix}3\\4\end{bmatrix}$. Then find the other one.

② Is $\begin{bmatrix}0 & -1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$ unitary? Yes.

③ Find a subset of the columns of

$$A=\begin{bmatrix}2 & 4 & -3 & -9 & -1 & -17\\ 3 & -6 & 13 & 56 & 3 & 87\\ 1 & -2 & 3 & 12 & 1 & 21\end{bmatrix}$$

that is linearly independent and spans $C(A)$, using

$$A \xrightarrow{\text{RREF}} \begin{bmatrix}1 & 2 & 0 & 3 & 0 & 4\\ 0 & 0 & 1 & 5 & 0 & 6\\ 0 & 0 & 0 & 0 & 1 & 7\end{bmatrix}$$

$$\begin{array}{cccccc}P & & P & & P & \end{array}$$

1. $3^2+4^2=5^2$

$x^2+y^2=r^2$

$\cos\theta = \frac{x}{r} = \frac{3}{5}$

$\sin\theta = \frac{y}{r} = \frac{4}{5}$

$\underbrace{\begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}}_{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}, \quad \underbrace{\begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix}}_{\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}}$

rotation reflection

Day 12

2. $$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}^* = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 \Rightarrow \text{It's unitary.}$$

3. $$\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 13 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \right\}$$

Note: ___, ___, ___ is a list;

{___, ___, ___} is a set.

(4) Find a finite set of vectors that is lin. indep. and whose span equals

$$\left\langle \left\{ \begin{bmatrix} 0\\0\\0\\0 \end{bmatrix}, \underset{P}{\begin{bmatrix} 1\\0\\0\\0 \end{bmatrix}}, \underset{P}{\begin{bmatrix} 0\\1\\0\\0 \end{bmatrix}}, \begin{bmatrix} 3\\2\\0\\0 \end{bmatrix}, \begin{bmatrix} 7\\9\\0\\0 \end{bmatrix}, \underset{P}{\begin{bmatrix} 0\\0\\1\\0 \end{bmatrix}} \right\} \right\rangle .$$

(5) Find an indep. spanning set of

$$\left\langle \left\{ \begin{bmatrix} 1\\0\\-1\\4 \end{bmatrix}, \begin{bmatrix} -1\\-6\\11\\0 \end{bmatrix}, \begin{bmatrix} 3\\16\\-29\\-2 \end{bmatrix}, \begin{bmatrix} 0\\4\\-6\\2 \end{bmatrix} \right\} \right\rangle$$ using

$$\begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 4 & -6 & 16 \\ -1 & -6 & 11 & -29 \\ 4 & 2 & 0 & -2 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccc} P & P & P & \end{array} \end{array}$$

(See solutions two pages down.)

Day 13

① $A = \begin{bmatrix} 1 & 2 & 10 & 13 & -3 \\ 2 & 5 & 24 & 31 & -8 \\ -3 & -8 & -38 & -49 & 14 \\ 1 & 1 & 6 & 8 & -2 \end{bmatrix}$

$$A^t \xrightarrow{RREF} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} P & P & P & \end{matrix} \end{array}$$

A^t, not A!

$$\overbrace{\begin{aligned} x_1 \qquad\qquad\quad + 4x_4 &= 0 \\ x_2 \qquad\; - 3x_4 &= 0 \\ x_3 - x_4 &= 0 \\ x_4 &= x_4 \end{aligned}}^{N(A^t)}$$

- $R(A)$ is spanned by a set of <u>three</u> vectors. Find such a set.
- $C(A)$ is also spanned by a set of <u>three</u> vectors. Find such a set. Hint: $C(A) = R(A^t)$.
- Find one vector that spans $L(A)$.

Day 12

(4) Put the columns together:

$$\begin{array}{c} \begin{bmatrix} 0 & 1 & 0 & 3 & 7 & 0 \\ 0 & 0 & 1 & 2 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccccc} & P & P & & & P \end{array} \end{array}$$ (already RREF)

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

(5) $$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -6 \\ 11 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 16 \\ -29 \\ -2 \end{bmatrix} \right\}$$

---

(1) Day 13

- $R(A) = C(A^t)$ is spanned by $\underbrace{\text{columns } 1, 2, 3}_{\text{pivots of } R \text{ where } A^t \xrightarrow[\text{RREF}]{} R}$ of A^t:

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 10 \\ 13 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 24 \\ 31 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ -8 \\ -38 \\ -49 \\ 14 \end{bmatrix} \right\}$$

- $C(A) = R(A^t)$ is spanned by the pivot rows of R, transposed:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

- $L(A) = N(A^t)$ is spanned by $\begin{bmatrix} -4 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

Day 13

1

Day 13

2 $B = \begin{bmatrix} 14 & 5 & 19 & -9 & -1 \\ 5 & 1 & 6 & -4 & 7 \\ -1 & 7 & 6 & 8 & -12 \\ 0 & -2 & -2 & -2 & 1 \end{bmatrix}$

$$[B^t \mid I_5] \xrightarrow{\text{RREF}} \left[\begin{array}{cccc|ccccc} 1 & 0 & 0 & .112 & 0 & 0 & .0064 & -.0912 & -.0576 \\ 0 & 1 & 0 & -.376 & 0 & 0 & .0928 & .1776 & .1648 \\ 0 & 0 & 1 & -.312 & 0 & 0 & .0536 & .1112 & .0176 \\ 0 & 0 & 0 & 0 & 1 & 0 & -.5 & .5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -.5 & -.5 & 0 \end{array}\right]$$

$\begin{array}{ccc} P & P & P \end{array}$

Find independent spanning sets for each of

$C(B)$, $R(B)$, $L(B)$, $N(B)$.

② The left four columns of the RREF matrix are the RREF of B^t. These tell us about $C(B)$, $R(B)$, and $\mathbf{L}(B)$ like in ①: Day 13

$$\underbrace{C(B)}_{R(B^t)} = \left\langle \underbrace{\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ .112 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -.376 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -.312 \end{bmatrix} \right\}}_{\text{linearly independent spanning set} = \{\ldots\}} \right\rangle$$

$$\underbrace{R(B)}_{C(B^t)} = \left\langle \underbrace{\left\{ \begin{bmatrix} 14 \\ 5 \\ 19 \\ -9 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 6 \\ -4 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ 7 \\ 6 \\ 8 \\ -12 \end{bmatrix} \right\}}_{\text{linearly independent spanning set} = \{\ldots\}} \right\rangle$$

$$\underbrace{L(B)}_{N(B^t)} = \left\langle \underbrace{\left\{ \begin{bmatrix} -.112 \\ .376 \\ .312 \\ 1 \end{bmatrix} \right\}}_{\text{linearly independent spanning set} = \{\ldots\}} \right\rangle$$

Day 13

For an indep. spanning set of N(B), take the non-pivot rows of the 5x9 RREF matrix, remove the left 4 columns, and finally transpose:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -.5 \\ .5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -.5 \\ -.5 \\ 0 \end{bmatrix} \right\}$$